Test for Trading Costs Effect in a Portfolio Selection Problem with Recursive Utility*

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Abstract

This paper addresses a portfolio selection problem with trading costs on stock market. More precisely, we develop a simple GMM-based test procedure to test the significance of trading costs effect in the economy regardless of the form of the transaction cost. We also propose a two-step procedure to test overidentifying restrictions in our GMM estimation. In an empirical analysis, we apply our test procedures to the class of anomalies used in Novy-Marx and Velikov (2016). We show that transaction costs have a significant effect on investors behavior for most of anomalies. In that case, investors significantly improve the out-of-sample performance of their portfolios by accounting for trading costs.

Keywords: Portfolio selection, test for trading costs effect, testing overidentifying restrictions, recursive utility

JEL classification: C12, C52, G11

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1 Introduction

The problem of optimal allocation of economic resources is far from being a recent issue. It is a problem which already existed in the world before the first century[1]. Nonetheless, there has been a strong advances in the literature of the optimal allocation of financial resources since the 20th century with the proposal of several strategies for portfolio selection, especially with the seminal work of Markowitz (1952) which offers an essential basis to portfolio selection in a single period. However, his quadratic form utility function hypothesis has been strongly criticized and many alternative utility functions such as power utility and exponential utility have emerged in the literature of portfolio optimization. Moreover, Epstein and Zin (1989, 1991) develop a more flexible version of the basic power utility model. This new version of utility retains the desirable scale-independence of the power utility[2] but breaks the link between the elasticity of intertemporal substitution and the coefficient of relative risk aversion. Regarding the large advantages of this class of preferences and their ability to explain financial variables, we use recursive utility to characterize investors' preferences in our economy. Hence, our work is related to the previous literature of portfolio optimization with recursive preferences (see Campbell and Viceira (2002), Campbell, Chacko, Rodriguez, and Viceira (2004),Campani, Garcia, and Lioui (2015)). More importantly, all those studies are carried out in a frictionless framework. Nonetheless, financial frictions in the form of liquidity costs, taxes, and transaction costs may affect investors' behavior on the financial market. For instance, an investor will have an incentive to invest in a more liquid asset compared to a less liquid asset. Indeed, according to Acharya and Pedersen (2005) the wealth problem which arises in the financial market due to the low market return at a given time can be amplified if selling investors hold illiquid assets at this time. In fact, the asset illiquid[3] could be seen as the potential loss because one cannot sell it at the price previously thought at a short notice. Moreover, investors will tend to have high preference for assets which require less costs to be invested in. Therefore, one needs to examine the rule played by those frictions in a portfolio selection problem with recursive preferences. We address this issue by treating trading costs as the only friction in the financial market since

[1]For instance, in circa 400 A.D. Rabbi Issac Bar Aha recommended that one should always divide his wealth equally into three parts: land, merchandise and cash at hand.

[2]such as the relative risk aversion coefficient and the elasticity of intertemporal substitution are constant

[3]See Amihud (2002) for a more general definition of the asset illiquidity.

assets illiquidity costs could also be seen as a certain transaction cost (Acharya and Pedersen (2005)). Our paper is then related to the vast literature about transaction costs and portfolio selection problems (see Dumas and Luciano (1991), Lynch and Balduzzi (1999), Lynch and Balduzzi (2000), Liu and Loewenstein (2002), Liu (2004), Lesmond, Schill, and Zhou (2004), Buss, Uppal, and Vilkov (2011), Gârleanu and Pedersen (2013), Novy-Marx and Velikov (2016), He and Modest (1995), Detemple and Rindisbacher (2005), Schroder and Skiadas (2005) among others). However, most of the studies in the literature about trading costs effect depend largely on the form of the frictions assumed in the model. Indeed, with proportional or fixed costs, the optimal investment policy is shown to be in the form of a no-trade region so that trade occurs only when the proportion of wealth invested in the risky asset is outside this region (Dumas and Luciano (1991), Lynch and Balduzzi (1999), Lynch and Balduzzi (2000), Liu and Loewenstein (2002), Liu (2004), Buss, Uppal, and Vilkov (2011)). Nevertheless, the optimal investment policy is no longer in the form of a no-trade region with quadratic trading costs since the investor trades at each period in small quantities (Heaton and Lucas (1996), Gârleanu and Pedersen (2013)). Moreover, Lynch and Balduzzi (1999) compute the utility cost due to the presence of these frictions and obtain an utility cost close to 4% with proportional costs and about 15% when added fixed costs to the proportional one.

In this paper, to overcome this problem, we develop a simple test procedure which allows us to test the significance of trading costs effect on a given asset in the economy without any assumption about the form of these frictions (besides being non-decreasing). The most interesting property of this test procedure is that our results do not depend on the form of the trading costs in our model. To our knowledge, this paper seems to be the first one to propose a statistical test for trading costs effect in the context of portfolio selection. Our test boils down to testing the nullity of a parameter which is at the boundary of the parameter space under the null. Its asymptotic distribution is non standard and is derived using results by Andrews (1999). In the empirical application, we apply our test procedure to the class of anomalies used in Novy-Marx and Velikov (2016). We obtain that transaction costs have significant effect for most of anomalies considered in particular those whose trading costs exceed 1% of the gross return. Not surprisingly, trading costs do not have a significant effect when the risky asset is assumed to be the market portfolio.

Our test procedure relies on the assumption that the model is correctly specified. We wish to test this assumption using Hansen's J-test for overidentifying restrictions. However, when the true parameter is close to the boundary of the parameter space, the standard J-test based on the χ^2 critical value suffers from overrejection. To overcome this problem, we propose a two-step procedure to test overidentifying restrictions when the the parameter of interest approaches the boundary of the parameter space. This paper is related to the work of Ketz (2019) who proposes a J-test based on adjusted critical values and a modified J-test. We find by simulations that our two-step procedure has good small sample properties.

By proposing a simple test procedure to evaluate the effect of transaction costs in the investment process based on a conditional moment inequality, this paper is also related to the vast literature on inference using moment inequalities (see Andrews and Guggenberger (2009), Andrews and Barwick (2012), Andrews and Soares (2010), Bugni (2010) among others). These papers are concerned with set identification while here our parameters are point identified. Moreover, we are interested in estimating the slackness of the moment inequality, this slackness having an interpretation in terms of transaction cost. Our approach is related to Moon and Schorfheide (2009) who exploit inequality moment conditions to identify some economically relevant parameters and Romano, Shaikh, and Wolf (2014) who propose a two step approach to test a finite number of moment inequalities.

Finally, we measure the economic gain from taking transaction costs into account compared to strategy that ignores transaction costs, assuming proportional trading costs and comparing the out-of-sample performance of the selected portfolios. For this purpose, we use several statistics such as the certainty equivalent (CE), the Sharpe ratio (SR) and the portfolio mean. We find that a rational investor who takes transaction costs into account outperforms a naive investor for assets whose trading costs have been shown to have significant effect according to our test procedure.

The rest of the paper is organized as follows. The model economy and the first order conditions from optimization problem are presented in Section 2. In Section 3, we develop a GMM-based test procedure to test whether trading costs have a significant effect. A two-step procedure for testing overidentifying restrictions in the GMM estimation is proposed in Section 4. Section 5 presents the empirical analysis where the test developed in Section 3 is applied to the twenty-three anomalies used in Novy-Marx and Velikov (2016). In Section 6, we evaluate the economic gain from taking transaction costs into account. Our conclusion and remarks are presented in Section 7. Proofs and tables are collected in Appendix.

2 The model and the first order conditions for the optimization problem

In this section we will start by the model economy before talking about the optimization problem.

2.1 The model

We consider a simple economy with two assets in which an investor can trade:

1. One risk-free asset (a bond) with a constant rate R^f. In general R^f will be calibrated to be the mean of the one-month Treasury-Bill rate observed in a monthly data.

2. One risky asset with a gross return R_{t+1} assumed to be predictable using the information available at period t.

We consider a finite-life horizon investor with recursive preferences as introduced in Epstein and Zin (1989, 1991). The investor's utility function is defined recursively by the following equation:

$$U_t = \left[(1-\beta)C_t^{\rho} + \beta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{\rho}{1-\gamma}}\right]^{\frac{1}{\rho}} \tag{1}$$

where U_t is the utility level at time t which is a function of the current consumption C_t and the future expected utility given time t information, E_t denotes the conditional expectation given the information available to agents at time t. $\beta \in (0,1)$ is the rate of time preferences, γ is the coefficient of relative risk aversion which controls for investor's attitude over the states of the economy. $\Psi = \frac{1}{1-\rho}$ controls for intertemporal consumption allocation and will be considered as a measure of the elasticity of intertemporal substitution (EIS).

Recursive utilities help us to distinguish the relative risk aversion from the elasticity of intertemporal substitution. This property of separability of these two parameters is very useful when one is interested in a portfolio selection problem (see Campani, Garcia, and Lioui (2015)). According to Campani, Garcia, and Lioui (2015), it is observed that investors tend to take more risk for greater values of the EIS and the optimal investment decision is more affected by the EIS than the relative risk aversion.

Because investors in general face some frictions such as liquidity costs, taxes, transaction costs, which can affect their behavior on financial market, it is important to incorporate these frictions when one is interested in a portfolio selection problem. For instance, Dumas and Luciano (1991), Lynch and Balduzzi (1999), Lynch and Balduzzi (2000), Liu and Loewenstein (2002), Liu (2004) show that realistic proportional or fixed costs cause optimal portfolio rebalancing frequency to decline considerably. Lesmond, Schill, and Zhou (2004) also argue that the large gross spreads observed on momentum trades creates an "illusion of profit opportunity when in fact, none exists" because of the presence of trading costs. The same argument has been pointed out by Novy-Marx and Velikov (2016) who show that with trading costs in financial market, a strategy can have a significant positive alpha relative to the explanatory assets without significantly improving the investment opportunity set. Therefore, it is important not to ignore trading costs when one is particularly interested on investors behavior on financial markets. Hence, we assume that investors face transaction costs when trading on the risky asset and the transaction costs are assumed to be the only source of frictions in the financial market. In fact, unlike in He and Modest (1995) who consider four types of market frictions[4], we assume that the transaction costs are the only source of friction in our economy. Moreover, since the transaction costs could take several forms[5], we do not assume any form to these costs when implementing our test as in the previous literature. Trading[6] costs could be seen as all costs incurred by investors in the process of buying or selling an asset on the stock market. Hence, trading costs include brokerage fees, cost of analysis, information cost and any expenses incurred in the process of deciding upon and placing an order. Delay in execution which cause prices at which one trades to be different from those at which one planned to trade may be included as well.

Let y_t denote the proportion of the risky-asset that the investor holds at time t in the share

[4] A no-short-sale constraint, a borrowing constraint, solvency constraints and the transaction costs.

[5] Fixed costs, proportional costs, a combination of the fixed and proportional costs, the quadratic costs.

[6] We use the terms "trading cost" and " transaction cost" interchangeably.

of portfolio value. A portfolio will be defined as a list of weights y_t, $1 - y_t$ that represents the amount of capital invested in the risky asset and the bond respectively. Hence, the return on the portfolio is given by:

$$R_{p,t+1} = y_t R_{t+1} + (1 - y_t) R^f \tag{2}$$

We also assume that at each period of time the investor consumes a fraction of his current income. Thus, if A_t is his wealth at time t and C_t the consumption level then we define $k_t = \frac{C_t}{A_t}$ so that k_t varies (is random) as in Lynch and Balduzzi (2000). This assumption is more realistic than the one in Campbell and Viceira (2002) who assume a constant consumption-wealth ratio over time $\frac{C_t}{A_t} = k$.

In our model, the investor does not receive labor income, so he finances consumption entirely from financial wealth. Indeed, an external source of income to the financial market could affect investors' behavior toward risk and bias transaction costs effect on a portfolio selection problem as well as the result of our test procedure. Hence, assuming only the financial wealth in the model is a convenient assumption when one is interested in trading costs effect.

Let T_t denote the transaction costs faced by the investor at the time t. Therefore, the law of motion of his total wealth is

$$C_t + A_{t+1} = R_{p,t} A_t - T_t \tag{3}$$

where $R_{p,t}$ is the gross return on the portfolio at time $t-1$. The right hand side of this equation represents the available resources for the investor at time t. These resources are used to finance consumption and saving for the next period.

We add the following constraint on the transaction cost.

$$T_t \leq f(y_t) \tag{4}$$

where f is a non-decreasing function of the share of risky-asset held by the investor at time t in his portfolio. The constraint (4) is reasonable as it covers various types of transaction costs (constant, proportional, quadratic among others). We distinguish two types of investors. On the one hand, a naive investor is an agent who does not take into account the constraint (4) in his optimization problem. This agent does not know the form of f and ignores that his decision investment will influence the level of the transaction costs. Therefore, he takes the transaction costs as given and does not care about (4) in his investment process. On the other hand, a rational investor is an agent who has a clear idea about how his decision could affect the transaction costs through (4). He made the effort to learn the function f and takes into account the constraint (4) in his investment process. In the next section, we derive the Euler equations obtained by a rational investor.

2.2 First-order conditions for consumption-investment optimization problem

To derive the first order conditions for the optimization problem, let us first observe that the utility function U_t in (1) is a function of C_t and U_{t+1}. Moreover, U_t is homogeneous of degree 1 in its two arguments that is

$$U_t(aC_t, aU_{t+1}) = aU_t(C_t, U_{t+1})$$

for every positive value a. Hence, by the Euler's theorem, we have that

$$U_t = \frac{\partial U_t}{\partial C_t} C_t + E_t\left[\frac{\partial U_t}{\partial U_{t+1}} U_{t+1}\right]$$

where $\frac{\partial U_t}{\partial C_t}$ is the partial derivative of U_t with respect to C_t. Let us denote $U_1(t) = \frac{\partial U_t}{\partial C_t}$ and $U_2(t) = \frac{\partial U_t}{\partial U_{t+1}}$. Then

$$\frac{U_t}{U_1(t)} = C_t + E_t\left[\frac{U_2(t)}{U_1(t)} U_1(t+1) \frac{U_{t+1}}{U_1(t+1)}\right].$$

By definition $\frac{U_1(t+1)}{U_1(t)} U_2(t)$ is called the stochastic discount factor (SDF). It is convenient to take the SDF as given for the moment. We will later show (see Appendix A1 Equation 38) what the SDF based on the recursive preferences is. Let us denote $J_t = \frac{U_t}{U_1(t)}$ and $\frac{M_{t+1}}{M_t} = \frac{U_1(t+1)}{U_1(t)} U_2(t)$. We can rewrite Equation (1) as follows

$$J_t = C_t + E_t\left[\frac{M_{t+1}}{M_t} J_{t+1}\right] \tag{5}$$

In the sequel, we will maximize (5) instead of (1). Hence, the planner maximizes (5) subject to (2)-(4). More precisely, the new optimization problem is given by

$$J_t(A_t, y_{t-1}, I_t) = \max_{C_t \geq 0, A_{t+1}, y_t} \left\{ C_t + E_t \left[\frac{M_{t+1}}{M_t} J_{t+1}(A_{t+1}, y_t, I_{t+1}) \right] \right\} \tag{6}$$

$$C_t + A_{t+1} = R_{p,t} A_t - T_t \tag{7}$$

$$T_t \leq f(y_t) \tag{8}$$

By solving this new optimization problem (see Appendix A1 for more details about the resolution of this problem), we obtain the following first-order conditions for the consumption-investment optimization problem

$$E_t \left[\beta^{\frac{\lambda}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}} \right] = 1 \tag{9}$$

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} \left(R_{t+1} - R^f \right) \right] \leq 0 \tag{10}$$

for $t = 1, ..., T-1$ where $R_{p,t+1}$ is the gross return on the selected portfolio and given by Equation (2). Moreover, if the trading costs are not present or if they are fixed regardless of y_t (case wher $f'(y_t)$) then (10) becomes

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} \left(R_{t+1} - R^f \right) \right] = 0 \tag{11}$$

The relation in (10) holds for any non-decreasing function f. Hence, it is true for proportional and quadratic transaction costs. Note that a naive investor who takes the transaction costs as given will base his decision on (9) (11) instead of (9) (10) and hence his decision might be suboptimal.

The system of Euler Equation (9) and Inequality (10) are going to be used as a set of moment conditions to estimate the parameters of the model and to construct tests.

3 Testing trading cost effect using GMM estimation

Our goal in this section is to develop a GMM-based test procedure which allows us to test the significance of the transaction costs effect in the economy.

3.1 The GMM procedure to estimate the parameter of interest

To test if trading costs have a significant effect on a given asset, we first transform the first order conditions obtained in (9) and (10) as moment equalities. Let us first look at (10). According to (10) and (34) in Appendix A1, there exists a non-negative stochastic process δ_t such that

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}\left(R_{t+1}-R^f\right)\right]+\delta_t=0 \tag{12}$$

Since δ_t is a stochastic process, this time varying parameter is not identified. However, its sign is identified as we are going to see below. Consider a vector of instruments x_t which belong to the information set available to the investors at time t. This vector contains typically the lagged values of C_{t+1}/C_t and of the Market returns. These variables are all positive. We multiply x_t on both sides of Equation (12) to obtain an unconditional moment condition:

$$E\left[\left\{\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}\left(R_{t+1}-R^f\right)+\delta_t\right\}x_t\right]=0. \tag{13}$$

Now let us denote[7] $\delta = E\left(\delta_t x_t\right)/E\left(x_t\right) \geq 0$. Then, the moment condition (13) can be written

$$E\left[\left\{\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}\left(R_{t+1}-R^f\right)+\delta\right\}x_t\right]=0 \tag{14}$$

So, the parameter δ can be estimated by the generalized method of moments (GMM) along with the other parameters (β, λ, ρ). Interestingly, by the positivity of x_t, $\delta = 0$ if and only if $\delta_t = 0$ with probability 1 and $\delta > 0$ if and only if $\delta_t > 0$ with probability one. So testing the nullity of δ gives us information of the nullity of δ_t and informs us on the effect of the transaction costs in the economy. Hence, the test procedure, we are going to propose in the next subsection, will be about the significance of the parameter δ. Thus, for a given risky asset in the economy, a significant parameter δ means that investors have to account for trading costs in this asset when including it in their portfolio. However, when δ is not statistically significant then trading costs could be ignored in the portfolio selection process without significant consequences in terms of utility loss.

[7] Using this notation, δ is a vector with the same dimension as x_t. In practice, we assume that δ is the same for each instrument. Moreover, if δ_t and x_t are uncorrelated, then $\delta = E(\delta_t)$.

Let

$$g(Z_t,\theta) = \begin{pmatrix} \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} R_{p,t+1}\right]^{\frac{\lambda}{\rho}} - 1 \\ \left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}\left(R_{t+1}-R^f\right)+\delta \end{pmatrix} \otimes \begin{pmatrix} 1 \\ x_t \end{pmatrix} \tag{15}$$

where $\theta = (\delta,\beta,\lambda/\rho,\rho)'$, $Z_t = \left\{\frac{C_{t+1}}{C_t}, R_{p,t+1}, R_{t+1}, x_t'\right\}$ where x_t is a vector of instruments, C_t the level of consumption at time t, $R_{p,t+1}$ the gross return on the portfolio, and R_t the gross return on the risky asset. Then, we use $E(g(Z_t,\theta)) = 0$ with g defined by (15) as the set of moment conditions to estimate $\theta = (\delta,\psi')'$ by a two-step GMM procedure where $\psi = (\beta,\lambda/\rho,\rho)'$ with β the discount factor, $\gamma = 1-\lambda$ the relative risk aversion coefficient, and $EIS = \frac{1}{1-\rho}$. In our test procedure ψ will be treated as a vector of identified nuisance parameters. Let $G_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g(Z_t,\theta)$ denote the empirical counterpart of the moment conditions where T is the sample size and $l_T(\theta,\hat{W}) = -\frac{T}{2}G_T(\theta)'\hat{W}G_T(\theta)$ the GMM objective function where $\hat{W}$ is a random symmetric positive definite matrix such that $\hat{W} \xrightarrow{P} W$ with W a non-random symmetric positive definite matrix.

Let $\hat{\theta}$ denote the two-step GMM estimator of θ using $E(g(Z_t,\theta)) = 0$ as the set of moment conditions. We obtain this estimator using the following procedure.

First, we estimate θ by GMM using $W = I$ so that we obtain the first step estimator, $\hat{\theta}(I) = \text{argmax}_\theta\, l_T(\theta, I)$.

We then estimate $S = E(g(Z_t,\theta_0)g(Z_t,\theta_0)')$ by $\hat{S} = \frac{1}{T}\sum_{t=1}^{T} g_t(\hat{\theta}(I))g_t(\hat{\theta}(I))'$ where $g_t(\theta) = g(Z_t,\theta)$ so that the second step GMM estimator is given by $\hat{\theta} = \text{argmax}_\theta\, l_T(\theta,\hat{S}^{-1})$.

3.2 Testing the significance of the transaction cost effect

Our objective in this part is to propose a procedure to test whether the transaction costs have a significant effect on investor's welfare (in terms of utility loss) based on the two-step GMM estimation presented above.

An interesting property of this test procedure is that our results do not depend on any form given to trading costs in the model as it has been done in the literature. Indeed, the conclusions of most of the studies in the literature about trading costs effect depend largely on the form of frictions assumed in the model. To test whether trading costs have a significant effect, we formulate the following hypothesis:

$H_0 : \delta = 0$ vs $H_1 : \delta > 0$

where $\delta \in \mathbb{R}^+$ is the parameter which informs us about the transaction cost effect in our economy. Using a compact form, the test hypothesis becomes:

$H_0 : H\theta = 0$ vs $H_1 : H\theta > 0$

where $H = (1, 0, 0, 0)$ and $\theta = (\delta, \beta, \lambda/\rho, \rho)'$ the vector of parameters to be estimated by GMM.

To implement this test, one needs to derive the asymptotic distribution for $\hat{\delta}$ under the null hypothesis. Let us first introduce some useful notations.

Notations.

Let K be the number of moment conditions, $G(\theta) = E(g(Z_t, \theta))$ and $\Gamma = \frac{\partial G(\theta_0)}{\partial \theta'}$ be the $K \times 4$ matrix of right partial derivatives of $G(\theta)$ at θ_0. Let $l_T(\theta) = -TG_T(\theta)' \widehat{S}^{-1} G_T(\theta)/2$ and $\hat{\theta} = \arg\max_{\theta \in \Theta} l_T(\theta)$.

To derive asymptotic distributions under the null hypothesis, we also need a set of assumptions.

Assumption A.

1. $Z_t = \left\{ \frac{C_{t+1}}{C_t}, R_{p,t+1}, R_{t+1}, x_t' \right\}$ is a stationary and ergodic process.
2. $\theta_0 \in \Theta = \left\{ \theta \in \mathbb{R}^4 : \theta = (\delta, \beta, \lambda/\rho, \rho)', \delta \geq 0, 0 \leq \beta \leq 1, \|\theta_j\| \leq M_j,\ j \leq 4 \right\}$.
3. Identification: $G(\theta) = 0$ if and only if $\theta = \theta_0$.
4. Dominance: (i) $E(\sup_{\Theta} \|g(Z_t, \theta)\|) < \infty$

   (ii) $E(\sup_{\mathcal{N}} \|\frac{\partial g(Z_t,\theta)}{\partial \theta'}\|) < \infty$ where $\mathcal{N}$ is a neighborhood of θ_0 and $\frac{\partial g(Z_t,\theta)}{\partial \theta'}$ denotes the $K \times 4$ matrix of right partial derivatives of $g(Z_t, \theta)$.
5. $\widehat{S} \xrightarrow{P} S$ where $S = E(g(Z_t, \theta_0) g(Z_t, \theta_0)')$ is a finite positive definite matrix.
6. Γ is full column rank.

Assumption A1 is a standard assumption in macroeconometrics. Assumption A2 proposes a reparametrization of the model so that the resulting moment condition $g(Z_t, \theta)$ is continuous in θ, moreover Θ is assumed to be compact which guarantees the consistency of the GMM estimator. The other assumptions are standard and can be found in textbooks (see for instance Hayashi (2000)) except that g is not assumed to be differentiable for all $\theta \in \Theta$ but only right differentiable.

A standard and convenient assumption in the literature is that the true parameter θ_0 is an interior of the parameter space. Indeed, it allows the use of the mean value theorem useful to establish the asymptotic normality of $\hat{\theta}$. When the true parameter θ_0 is an interior point of Θ and Assumption A is satisfied, the following results hold (see Hayashi (2000)):

- $\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \left(\Gamma' S^{-1} \Gamma\right)^{-1})$.
- $Wald_{stat} = \frac{T(\hat{\delta}-\delta_0)^2}{\widehat{\sigma}_\delta^2} \xrightarrow{\mathcal{L}} \chi^2(1)$, where $\widehat{\sigma}_\delta^2$ is a consistent estimator of $\sigma_\delta^2 = H\left(\Gamma' S^{-1} \Gamma\right)^{-1} H'$.
- $J = TG_T(\hat{\theta})' \widehat{S}^{-1} G_T(\hat{\theta}) \xrightarrow{\mathcal{L}} \chi^2(K-4)$ where K is the number of moment conditions and 4 the number of estimated parameters.

However, in our economic application, the true parameter θ_0 is not an interior point of Θ under the null hypothesis $H_0 : \delta = 0$. When the true parameter is on the boundary, the asymptotic distribution of $\hat{\theta}$ is no longer a standard distribution (see Andrews (1999)). The following proposition establishes the asymptotic distribution of the Wald test statistic under the null hypothesis.

Proposition 1 *Let $\widehat{\sigma}_\delta^2$ denote a consistent estimator of the asymptotic variance of $\hat{\delta}$. Assume that Assumption A holds and that θ_0 is such that $\delta = 0$* and $(\theta_2, \theta_3, \theta_4)$ are interior points of the parameter space. *Then,*

$$W = \frac{T\hat{\delta}^2}{\widehat{\sigma}_\delta^2} \xrightarrow{\mathcal{L}} \frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$$

where $\chi^2(0)$ is the Dirac distribution at the origin and $\chi^2(1)$ is a chi-square distribution with one degree of freedom.

Remark: A consistent estimator for the asymptotic variance of $\hat{\delta}$ will be obtained based on a bootstrap method. As noted by Andrews (1999), the standard bootstrap does not generate consistent estimators of the asymptotic standard errors of extremum estimator when the true parameter is on the boundary. Hence, we use a version of the bootstrap procedure in which bootstrap samples of size T_1 $(< T)$ rather than T, are employed (for more details about this procedure, see Andrews (1999, p.1371)).

The asymptotic distribution of the Wald test under H_0 is a mixture of a chi-square with one degree of freedom and a mass-point at zero. Its critical values are 1.642, 2.706, and 5.412 for significance level 10%, 5% and 1% respectively, see Carrasco and Gregoir (2002). On the other hand when the true parameter is an interior point of the parameter space, the asymptotic distribution of the Wald test is a chi-square distribution with one degree of freedom $\chi^2(1)$ instead of a mixed distribution so that its critical values are given by 2.71, 3.84, and 6.63 for significance levels 10%, 5%, and 1%. We see that the correct critical values are smaller than those given by the $\chi^2(1)$, hence using mistakenly the $\chi^2(1)$ critical value would yield a test that lacks of power. To prove Proposition 1, we use results from Lemma 1 in Appendix A2. Its proof, given in Appendix A2, draws from results by Andrews (1999).

This test procedure is based on the GMM estimation of the parameters assuming the model is correctly specified. To test the validity of the moment conditions, it is customary to test overidentifying restrictions.

4 Testing overidentifying restrictions

In this section we are going to propose a two-step procedure which helps us to test overidentifying restrictions when one component of the parameter of interest may be at the boundary of its parameter space.

4.1 J-test when the true parameter is near or at the boundary of the parameter space

When the number of moment conditions exceeds the number of unknown parameters to be estimated by GMM, one can test the model validity by testing overidentifying restrictions. A common test used for this purpose is the J-test proposed by Hansen (1982) and one of the assumptions underlying this test is that the true parameter is an interior point of the parameter space. In this situation, Hansen's J-statistic satisfies $J = TG_T(\hat{\theta})'\hat{S}^{-1}G_T(\hat{\theta}) \xrightarrow{\mathcal{L}} \chi^2(K-L)$ where K is the number of moment conditions and L the number of estimated parameters. However, when the true parameter is on the boundary of the parameter space, Ketz (2019) shows that the standard J-test suffers from overrejection. In fact, when only one component of the parameter of interest is at the boundary of the parameter space, the asymptotic distribution of the J-statistic is $\tau\chi^2(K-L)+(1-\tau)\chi^2(K-L+1)$ with $\tau\in(0,1)$ which is a mixture of two independent chi-square distributions. In this situation, the standard J-test based on $\chi^2(K-L)$ suffers from overrejection because $\chi^2(K-L)$ is dominated by $\tau\chi^2(K-L)+(1-\tau)\chi^2(K-L+1)$. A simple way to control the nominal size of the J-test in such a situation is to use a conservative critical value based on $0.5\chi^2(K-L)+0.5\chi^2(K-L+1)$[8] since $\tau\chi^2(K-L)+(1-\tau)\chi^2(K-L+1)$ is dominated by $0.5\chi^2(K-L)+0.5\chi^2(K-L+1)$. Ketz (2019) (see Ketz (2017) for more details) also proposes a modified J-statistic which has the same asymptotic distribution as the standard J-test under the null hypothesis. The modified J-statistic is given by

$$J^M = 2T\hat{M}_T\left(\tilde{\theta}\right) \tag{16}$$

where

$$\begin{aligned}\hat{M}_T(\theta) &= G_T'\left(\hat{\theta}\right)\hat{W}G_T\left(\hat{\theta}\right)/2+G_T'\left(\hat{\theta}\right)\hat{W}\frac{\partial}{\partial\theta'}G_T'\left(\hat{\theta}\right)\left(\theta-\hat{\theta}\right)\\ &+ \left(\theta-\hat{\theta}\right)'\left(\frac{\partial}{\partial\theta'}G_T'\left(\hat{\theta}\right)\right)'\hat{W}\frac{\partial}{\partial\theta'}G_T'\left(\hat{\theta}\right)\left(\theta-\hat{\theta}\right)/2,\end{aligned}$$

$\hat{\theta}$ is the GMM estimator of θ and $\tilde{\theta}$ is the minimizer of $\hat{M}_T(\theta)$. Ketz (2019) shows that J^M is asymptotically distributed as a $\chi^2(K-L)$. However, in finite sample, both tests seem to lack of power in some directions (see Ketz (2017)). Therefore, we propose a simple two step procedure to test overidentifying restrictions.

4.2 A two-step procedure to test overidentifying restrictions

In this part of our analysis, we are going to propose a two-step method for testing overidentifying restrictions in our GMM estimation procedure. In the first step we will test the significance of δ based on a first step estimation. In the second step, we will use this information to decide whether to use the standard critical value or the adjusted critical value of Ketz (2019) to implement the J-test.

Nonetheless, to implement the test about the nuisance parameter in the first step, we need to have a first step consistent estimator of the parameter δ. So, we need some assumptions about the set of moment conditions used in our GMM estimation procedure.

[8] Critical values can be computed using the simple algorithm described in Appendix A4.

Let $g = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$ where $E\left[g(Z_t, \theta)\right] = 0$ is the set of moment conditions to be used in our estimation process, g_1 is a $k_1 \times 1$ vector and g_2 a $k_2 \times 1$ vector. Hence, we obtain that

$$E\left[g(Z_t, \theta)\right] = E\begin{pmatrix} g_1(Z_t, \theta) \\ g_2(Z_t, \theta) \end{pmatrix} = \begin{pmatrix} E\left[g_1\left(Z_t, \theta\right)\right] \\ E\left[g_2(Z_t, \theta)\right] \end{pmatrix}$$

To implement correctly our procedure, let us start by the following assumption.

Assumption B. $E\left[g_1(Z_t, \theta)\right] = 0$ if $\theta = \theta_0$ with $k_1 \geq L$ where L is the number of parameters to be estimated by GMM.

Assumption B implies that there is a subset of moment conditions which are correctly specified in order to identify the parameter θ so that we can obtain a consistent first step estimator of θ denoted by $\tilde{\theta}$ based only on $E\left[g_1(Z_t, \theta)\right]$. A similar assumption can be found in Moon and Schorfheide (2009), see their assumption 1(d). Using these two assumptions, we describe our procedure as follows:

Step 1: Test the following hypothesis about the unknown nuisance parameter δ:

H_0: $\delta = 0$ vs H_1: $\delta > 0$ at the significance level $\alpha_1 \in (0, 1)$. The test of this step is implemented based on the assumption B so that we can obtain a consistent estimator of θ using only $E\left[g_1(Z_t, \theta)\right] = 0$ as the set of moment conditions in the GMM process. The test statistic used to test the null hypothesis in this situation is the Wald test statistic given by:

$$W = \frac{T\tilde{\delta}^2}{\tilde{\sigma}_{\tilde{\delta}}^2}$$

where $\tilde{\sigma}_{\tilde{\delta}}^2$ is a consistent estimator of the asymptotic variance of $\tilde{\delta}$ and T is the number of observations used in the estimation process. Using the result of Proposition 1 under assumption A, we obtain that under the null hypothesis

$$W \xrightarrow{\mathcal{L}} \frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$$

where critical values have been given in Carrasco and Gregoir (2002) by 1.642, 2.706, and 5.412 for significant level 10%, 5%, and 1% respectively. We also simulate critical values for several significant level and report them in Table 12 in Appendix B.

Step 2: In this step we use the J-test to test overidentifying restrictions in our GMM estimation based on the entire available set of moment conditions $E\left[g(Z_t, \theta)\right] = 0$ at the significance level $\alpha_2 \in (0,1)$. In fact, as mentioned before when one element of the vector of parameters to be estimated by GMM is close to the boundary of its parameter space, the asymptotic distribution of the J-statistic could be different from the standard one depending on the value of this unknown nuisance parameter. Therefore, information obtained at the first step about the unknown nuisance parameter will be used to decide if we have to use the standard critical value or the adjusted critical value. Hence, if we denote by c_{α_1} the critical value of the test implemented in the first step then in the second step the J-test is implemented as follows:

- *If $W > c_{\alpha_1}$ then the critical value of the J-test is the standard one $\chi^2_{\alpha_2}(K-L)$*
- *If $W \leq c_{\alpha_1}$ then the critical value of the J-test is* $(0.5\chi^2(K-L) + 0.5\chi^2(K-L+1))_{\alpha_3}$

Let $c_{1\alpha_2}$ denote the $(1-\alpha_2)$- quantile of a $\chi^2(K-L)$ and $c_{2\alpha_3}$ the $(1-\alpha_3)$- quantile of the mixture $0.5\chi^2(K-L) + 0.5\chi^2(K-L+1)$.

Remark that $c_{2\alpha_3} > c_{1\alpha_3}$ because the distribution of $0.5\chi^2(K-L) + 0.5\chi^2(K-L+1)$ dominates that of $\chi^2(K-L)$.

Let H_0 be the null hypothesis that $E[g(Z_t,\theta)] = 0$ for some $\theta \in \Theta$.

The following proposition gives us an idea about the size of this procedure.

Proposition 2 *Assume that Assumption A holds. A two-step test for the null hypothesis H_0 which rejects H_0 when $W > c_{\alpha_1}$ and $J > c_{1\alpha_2}$ or when $W \leq c_{\alpha_1}$ and $J > c_{2\alpha_3}$ has an asymptotic size smaller than* $\max(\alpha_1,\alpha_2) + \alpha_3$.

The proof of Proposition 2 is given in Appendix A3. It takes into account the fact that the null hypothesis $\delta = 0$ may be rejected with probability α_1 even though δ is really equal to zero. Moreover, it does not require independence between W and J. Given we use upperbounds, it is expected that this test will be conservative.

The result in Proposition 2 implies in particular that our two step procedure controls size for all α_1, α_2, and α_3 chosen in such a way that $\max(\alpha_1,\alpha_2) + \alpha_3 \leq \alpha$ where $\alpha \in (0,1)$ is the global size of the J-test.

4.3 Simulations on our two-step procedure

Following Hansen and Singleton (1983), we consider a single-good economy of identical consumers, whose utility functions are

$$U(C_t) = \frac{C_t^\gamma}{\gamma} \tag{17}$$

where $\gamma < 1$ and C_t is aggregate real per capita consumption. We have 2 assets in the economy, R_{1t} denotes the return of the risk-free asset, whereas R_{2t} denotes the return of the risky asset for which some transaction costs may be relevant. Hence, the set of Euler Equations from the consumption-investment optimization problem are given by

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{1,t+1}\right] = 1 \tag{18}$$

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{2,t+1}\right] = 1-\delta \tag{19}$$

where δ is a positive constant. Let us denote by $X_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$, $r_{i,t+1} = \log(R_{i,t+1})$, $u_{i,t+1} = \log(U_{i,t+1})$ where $U_{i,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{i,t+1}$, $i = 1,2$. Let $Y_t = (X_t, r_{1,t}, r_{2,t})'$.

In our simulations, we will generate Y_t as a $VAR(2)$ process with Gaussian error. More precisely,

$$Y_t = a + BY_{t-1} + CY_{t-2} + \epsilon_t \tag{20}$$

$\epsilon_t \sim \mathcal{N}(0, \Sigma_\epsilon)$. At each iteration, the first two values of Y_t will be drawn using the marginal distribution of Y_t which is also a normal distribution. The other values of Y_t are then obtained using the dynamic in (20). Hansen and Singleton (1982, 1983) show that data generated according to (20) satisfy conditions (18) and (19). The orthogonality conditions in (18) and (19) will be used to form the moment conditions through the instruments $x_t = Y_{t-1}$ for estimating the parameter of interest $\theta = (\tilde{\gamma}, \beta, \delta)'$ by GMM where $\tilde{\gamma} = \gamma - 1$. In our simulations, to investigate the power of our test, we will replace (18) by

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{1,t+1}\right] = 1-\mu \tag{21}$$

where μ is a constant. When $\mu \neq 0$, the moment conditions based on (21) are not correctly specified. This particular case will be used to evaluate the power of our procedure under the alternative.

The parameters a, B, and C will be calibrated using monthly data from July 1973 to December 2013. The calibrated parameters are given in Appendix A5. We did our simulations for two different risky assets: the market portfolio (where δ may be 0) and the seasonality (where δ may be non zero because of transaction costs). The results of these simulations are summarized in Tables 1 and 2. These simulations are done with 100,000 replications. Columns 7, 8, and 9 contains the results about the J-test implemented using the standard critical value, the adjusted critical value, and the modified J-test proposed by Ketz (2019) respectively. Columns 2, 3, 4, 5, and 6 give the simulation results of our J-test implemented using a two-step procedure for various values of α_1, α_2, and α_3. The first row (case $\mu = 0$) corresponds to the size of the tests whereas the four other rows correspond to the power of the tests.

Table 1: **Empirical rejection rate when the risky asset is the market portfolio at the significant level** $\alpha = 5\%$

μ	Two step J-test					classical J-test	J-test with ACV	Modified J-test
	$\alpha_1 = 2.5$ $\alpha_2 = 2.5$ $\alpha_3 = 2.5$	$\alpha_1 = 4$ $\alpha_2 = 4$ $\alpha_3 = 1$	$\alpha_1 = 1$ $\alpha_2 = 1$ $\alpha_3 = 4$	$\alpha_1 = 4.7$ $\alpha_2 = 4.7$ $\alpha_3 = 0.3$	$\alpha_1 = 0.5$ $\alpha_2 = 0.5$ $\alpha_3 = 4.5$			
0.00	0.0321	0.0319	0.0462	0.0208	0.0502	0.0947	0.0505	0.0499
-0.1	0.4650	0.4826	0.7798	0.4420	0.8198	0.8270	0.7907	0.7864
-0.2	0.5400	0.5290	0.8290	0.5128	0.8423	0.8700	0.7981	0.8301
0.1	0.5340	0.5620	0.9650	0.5300	0.9980	0.9990	0.996	0.9948
0.2	0.6782	0.6580	0.9589	0.6420	0.9981	0.9987	0.9952	0.9960

Table 2: **Empirical rejection rate when the risky asset is the seasonality at the significant level** $\alpha = 5\%$

μ	Two step J-test					classical J-test	J-test with ACV	Modified J-test
	$\alpha_1 = 2.5$ $\alpha_2 = 2.5$ $\alpha_3 = 2.5$	$\alpha_1 = 4$ $\alpha_2 = 4$ $\alpha_3 = 1$	$\alpha_1 = 1$ $\alpha_2 = 1$ $\alpha_3 = 4$	$\alpha_1 = 4.7$ $\alpha_2 = 4.7$ $\alpha_3 = 0.3$	$\alpha_1 = 0.5$ $\alpha_2 = 0.5$ $\alpha_3 = 4.5$			
0.00	0.0318	0.0452	0.0458	0.0476	0.04985	0.0517	0.0358	0.0526
-0.1	0.6482	0.7938	0.7695	0.7980	0.8186	0.8212	0.7569	0.8197
-0.2	0.6780	0.7945	0.8214	0.8098	0.8387	0.8480	0.7892	0.8299
0.1	0.8945	0.9410	0.9200	0.9526	0.9789	0.9745	0.9190	0.9610
0.2	0.8820	0.9290	0.9519	0.9680	0.9979	0.9981	0.9500	0.9800

When the parameters are calibrated to match the market portfolio, we observe in the simulations that the parameter δ is statistically null for more than 84% of cases with an average value of $3.45e - 5$. Therefore, in this situation, the parameter δ is at the boundary of the parameter space. In Table 1, we observe that the classical J-test fails to control the size and suffers from over rejection. This result shows that at the boundary of the parameter space, the J-statistic using the $\chi^2(K - L)$ distribution as the asymptotic distribution over-rejects the null hypothesis. Nonetheless, for a wise choice of α_1, α_2, and α_3, our two-step procedure correctly controls the size with very good power. Moreover, our procedure out-performs the J-test based on the adjusted critical even if this test also controls the size in that case. Our test and the modified J-test perform similarly.

When the parameters are calibrated to match the seasonality as anomaly, simulations show significant δ with an average value of 0.0229. Table 2 shows that, in this situation, the standard J-test and the modified J-test slightly overreject under H_0. The J-test based on the adjusted critical underrejects under H_0. Our two-step procedure correctly controls the size for appropriate choice of α_1, α_2, and α_3 and gives most of the time better results than the modified J-test. We recommend using the values $\alpha_1 = \alpha_2 = 0.5\%$ and $\alpha_3 = 4.5\%$ as they seem to deliver the most powerful test.

In the next section we are going to apply our test procedures to empirical data.

5 Empirical Analysis

In this section we are going to apply empirically the test procedures described in Sections 3 and 4 in a context of portfolio selection with trading costs.

5.1 Data and data sources

In our empirical analysis, we use monthly data from July 1973 to December 2013. The monthly rate of the return on the value-weighted NYSE index is used as a proxy for the return on the market portfolio. The one-month Treasury-Bill (T-Bill) rate is used as a proxy for the risk-free rate and R^f is calibrated to be the mean of the one-month Treasury-Bill rate observed in the data. The consumption C_t is taken to be the U.S. real per capita consumption of non-durable goods and services, and is constructed using data from the Federal Reserve Bank of St Louis database. The monthly CPI inflation corresponding to the definition of the consumption adopted is also used to deflate the stock return and the risk-free rate. The return on the market portfolio and the interest rate are from the Fama-French database and the CPI are from the Federal Reserve Bank of St. Louis database. The returns on the risky assets (here anomalies) are from Robert Novy-Marx Data Library.

5.2 Descriptive statistics about assets returns

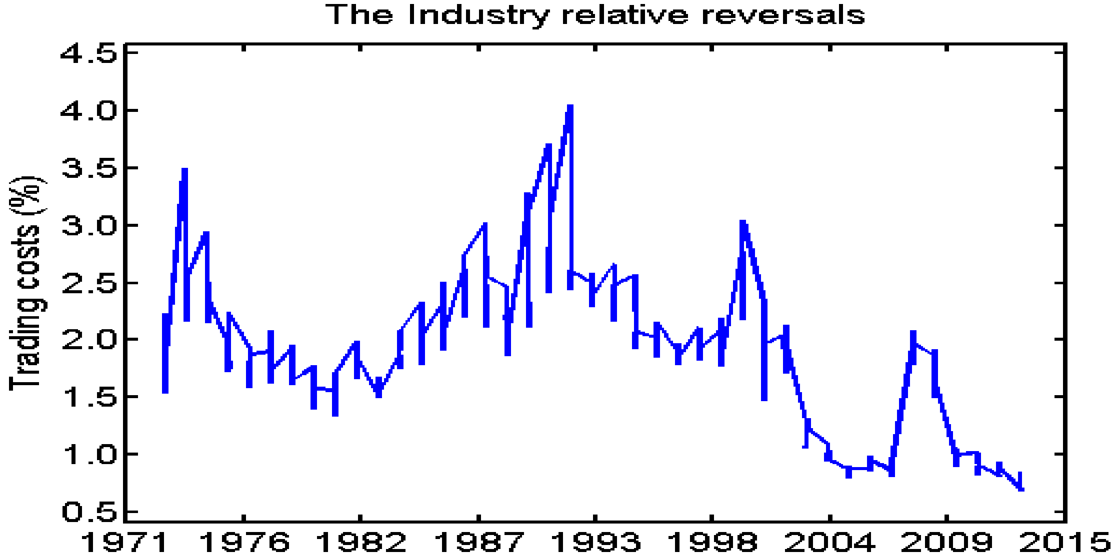
In this analysis we use two measures of per capita consumption. The first one is the expenditure on nondurable goods and the second one is the expenditure on nondurable goods and services. All the nominal variables such as nominal asset returns and nominal consumption are converted into real variables using the CPI inflation index corresponding to the definition of consumption adopted. The real consumptions are put into per capita terms using total civilian population from the Federal Reserve Bank of St. Louis database.

Figure 1 gives us the estimated trading costs on a specific anomaly namely the industry relative reversals from July 1973 to December 2013 based on data from Novy-Marx. Those trading costs are expressed as a percentage of the gross return. The average trading costs in this strategy is about 1.86% of the gross return with strong fluctuations between 1973 and 2013. As we can see it from the graph, these frictions represent an important part of the gross return. So, one should not ignore trading costs when investing in this asset.

Table 3 summarizes descriptive statistics on some variables of interest used in the estimation process. Columns 2 and 3 of this table contain the empirical mean of each variable in Column 1 and, in brackets, their empirical standard deviation. The difference between Columns 2 and 3 comes from the measure of CPI index (which changes with the measure of per capita consumption) used to transform nominal variables into real variables. Note that M_t is the real return on the market portfolio and c_{t+1}/c_t is the real consumption growth. For anomalies, we use the real returns net of transaction costs for assets whose trading costs exceed 0.50% of the gross return (see Table 10 in Appendix B).

Those descriptive statistics reveal that returns on stock market are substantially more volatile than the consumption growth and the bond market is very little volatile. Moreover, real returns appear to be relatively more stable when services are added to consumption measure indicating that the inflation index does not differ much across these measures of consumption.

Figure 1: **Trading costs in the portfolio based on the Industry relative reversals**



5.3 Estimation results and testing

We estimate the parameter $\theta = (\delta, \psi')'$ by GMM using Equation (15) and the set of instruments given by $x_{tl} = (1, \frac{c_t}{c_{t-1}}, ..., \frac{c_{t-l}}{c_{t-l-1}}, M_t, ..., M_{t-l})$. We consider $l = 1, 2, 3$ and two measures of per capita consumption. To test whether trading costs in a given strategy have a significant effect, we use the result of Proposition 1.

Since the main objective is to evaluate the effect of transaction costs, we report here the results about parameter δ, the other parameters are reported in Appendix B. Table 4 contains our estimation results when the consumption is measured by the nondurable goods. Table 5 provides results when the consumption is measured by the nondurable goods and services. The results of these two tables are obtained with $l = 2$. The results for $l = 1, 3$ are given in Tables 13 to 16 in Appendix B.

To test overidentifying restrictions, we use the two-step procedure proposed in Section 4. The p-value of this test is computed differently depending on the result of the first step procedure. If the parameter δ is significant at the first step at the significance level α_1 then the p-value is calculated using a chi-square distribution and is compared to α_2. But, if δ is not significant at the first step, the p-value is calculated using a mixture of independent chi-square

Table 3: **Descriptive statistics**

Variables	**Mean** **Nondurable and services**	**Mean** **Nondurable**
c_{t+1}/c_t	1.0009 (0.0039)	1.0006 (0.0072)
M_t	1.0082 (0.0488)	1.0149 (0.0811)
Bonds	1.0043 (0.0039)	1.0074 (0.0060)
Failure probability	0.9997 (0.0767)	1.0007 (0.1274)
Idiosyncratic volatility	0.9986 (0.0663)	0.9985 (0.1153)
Momentum	1.0046 (0.0643)	1.0080 (0.111)
PEAD (CAR3)	1.0035 (0.0283)	1.0059 (0.0486)
Industry momentum	0.9961 (0.0534)	0.9923 (0.0938)
Industry relative reversals	0.9948 (0.0424)	0.9892 (0.0718)
High frequency combo	1.0028 (0.033)	1.0031 (0.0578)
Short run reversals	0.9905 (0.0505)	0.9820 (0.0870)
Seasonality	0.9938 (0.0403)	0.9890 (0.0670)
Industry Relative Reversals (Low volatility)	1.0029 (0.0359)	1.0037 (0.0591)

distributions and is compared to α_3. We choose $\alpha_1 = \alpha_2 = 0.5\%$ and $\alpha_3 = 4.5\%$. This choice is motivated by the simulations of Section 4.

Column 2 in Tables 4 and 5 contains estimation results for various anomalies from Novy-Marx and Velikov (2016) using gross return and hence ignoring transaction costs. Quantities in brackets are statistics used to test whether δ is significant or not as in Proposition 1. These results show that the transaction costs are significant for most anomalies in particular all anomalies with trading costs exceeding 1% of their gross return (according to Novy-Marx and Velikov (2016)). Hence, for each of those strategies, the relation defined in (10) is satisfied with

Table 4: **GMM estimation result for testing trading costs effect**

Nondurable goods ($l = 2$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\hat{\delta}}$	J test	$\hat{\hat{\delta}}$	J test
Market Portfolio	1.3664e-05 (9.7245e-06)	9.028 (0.172)	1.0428e-05 (5.6759e-06)	9.057 (0.1704)
Size	0.0064013* (1.6657)	25.28+ (0.0003033)	0.0054747 (1.2442)	25.28+ (0.0003035)
Gross Profitability	0.0051534* (1.769)	4.775 (0.573)	0.0045914 (1.6171)	5.346 (0.5003)
Asset growth	3.5763e-13 (9.5228e-21)	7.674 (0.263)	5.1455e-13 (1.75e-20)	8.531 (0.2017)
Piotroski's F-score	0.0064663* (2.1903)	17.21+ (0.008531)	0.0042506 (0.97976)	17.41+ (0.007879)
PEAD (SUE)	0.0041693* (1.6842)	7.182 (0.3043)	6.5098e-13 (3.3749e-20)	9.515 (0.1466)
Industry Momentum	0.016371** (4.3862)	6.811 (0.3387)	1.3407e-14 (4.4593e-24)	10.35 (0.1106)
Industry Relative Reversals	0.029408** (5.312)	5.274 (0.5092)	3.2885e-13 (1.1809e-21)	1.827 (0.9349)
High Frequency Combo	0.0089251** (3.0787)	5.34 (0.501)	4.4611e-12 (3.4795e-19)	42.58+ (1.413e-07)
Short-run reversals	0.038244*** (5.8189)	5.787 (0.4475)	0.0041816 (0.55006)	6.299 (0.3906)
Seasonality	0.024196*** (10.0722)	12.43 (0.05295)	5.6079e-12 (2.0994e-18)	13.23+ (0.03953)
Industry Relative Reversals (Low volatility)	0.0090724** (3.7962)	3.284 (0.1936)	8.5873e-13 (3.272e-20)	23.09+ (0.0007676)

∗ 10%, ∗∗ 5%, ∗ ∗ ∗ 1%, + rejected at 5%

a strict inequality so that we obtain the following relation (see Appendix A6 for more details):

$$E_t \left\{ \beta^{\frac{\lambda}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} R_{jt+1} \right\} = 1 + \delta_t \omega_{it} \beta^{\frac{\lambda}{\rho}} \tag{22}$$

for $i, j = 1, 2$ with $i \neq j$ where δ is a positive parameter which shows us the effect of trading costs in the economy, $\omega_{1t} = -(1 - y_t)$, $\omega_{2t} = y_t$, $R_{1t+1} = R_f$, $R_{2t+1} = R_{t+1}$. Therefore, investors should incorporate these frictions when they have to participate to the financial market. Not surprisingly, the transaction costs are not significant for the market portfolio. We could explain this result essentially by the fact that trading costs on this asset are quite low so that the utility loss of not accounting for these frictions is negligible as well as the effect on the optimal

Table 5: **GMM estimation result for testing trading costs effect (continued)**

Nondurable goods and services ($l = 2$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}$	J test	$\hat{\delta}$	J test
Market Portfolio	1.0897e-12 (1.8736e-19)	10.16 (0.1181)	1.8e-12 (5.5e-19)	10.57 (0.1026)
Size	0.0028094 (1.1089)	18.46^{+} (0.00519)	0.0023344 (0.78437)	18.46^{+} (0.005173)
Gross Profitability	0.0028013 (1.3623)	5.574 (0.4726)	0.002528 (0.86494)	3.914 (0.6883)
Asset growth	0.0002716 (0.019201)	9.161 (0.1647)	6.9036e-13 (1.0931e-19)	9.387 (0.1529)
Piotroski's F-score	0.0036007* (2.1995)	16.83^{+} (0.009932)	0.0023875 (1.0298)	16.91^{+} (0.009616)
PEAD (SUE)	0.0025953* (1.9174)	6.897 (0.3305)	2.0229e-14 (9.8233e-23)	8.932 (0.1774)
Industry Momentum	0.0090454*** (5.6283)	10.48 (0.1057)	4.6873e-12 (1.7489e-18)	11.11 (0.08514)
Industry Relative Reversals	0.014861** (4.3446)	5.8 (0.446)	5.0304e-12 (3.7817e-18)	14.74^{+} (0.02239)
High Frequency Combo	0.0042342** (3.5827)	9.251 (0.1599)	7.6272e-12 (2.3702e-18)	48.91^{+} (7.757e-09)
Short-run reversals	0.020137* (1.8545)	2.385 (0.8811)	0.0018176 (0.41066)	8.809 (0.1846)
Seasonality	0.0124*** (8.0757)	11.9 (0.06428)	5.4295e-12 (5.9229e-18)	16.62^{+} (0.0108)
Industry Relative Reversals (Low volatility)	0.0039186* (1.8283)	5.975 (0.426)	2.0517e-13 (4.5296e-21)	26.92^{+} (0.0001496)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%

portfolio. So, if the market portfolio is used as the risky asset in our economy, the relation defined in (10) is satisfied with equality as in a frictionless setting. More precisely, in this situation, (10) becomes as follows

$$E_t \left\{ \beta^{\frac{\lambda}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} R_{jt+1} \right\} = 1 \tag{23}$$

for $j = 1, 2$. R_{jt+1} is the gross return of the asset j where $j = 1$ is the risk-free asset with $R_{1t+1} = R_f$ and $j = 2$ corresponds to the risky asset in the economy with $R_{2t+1} = R_{t+1}$. (23) comes from (22) with $\delta_t = 0$.

Most anomalies which exhibit a significant trading cost effect when the consumption is measured by the nondurable good have also significant trading costs effect with nondurable goods and services. Nonetheless, the intensity of the effect differs across these two measures of the consumption. Moreover, the number of models with significant trading cost effect tends to increase with the number of instruments (see Tables 13 and 14). In fact, when the number of instruments increases, estimation variance becomes smaller in such a way that the results of the tests become more accurate even if estimation bias could increase. But we can notice that estimation results are very similar across the set of instruments used in the estimation process. Hence, our results seem robust to the number of instruments used in the estimation process.

Column 4 in Tables 4 and 5 contains the results of the hypothetical case where net returns are used instead of gross returns[9]. In this case, the relation (10) should hold with equality. As expected, we find that the parameter δ is not significant for all anomalies confirming the validity of our procedure.

To test the assumption that our model is correctly specified, a two step J-test is implemented for each anomaly. The results of this analysis are reported in Columns 3 and 5 in Tables 4 and 5. According to these results, when transaction costs are ignored (the net return is not observed), the null hypothesis is rejected for only two anomalies : the size and the Piotroski's F-score. Hence, models based on these two anomalies as risky assets are not correctly specified. In the hypothetical case where the net returns are used, the null is rejected for five or more anomalies.

The estimates of the other parameters that characterize investors preferences are given in Tables 18 to 20 in Appendix. Standard errors are in brackets. We notice that, for most anomalies, the estimates of the preference parameters are significant. Using the result of this estimation, we can estimate the elasticity of intertemporal substitution by $\frac{1}{1-\hat{\rho}}$ which is in $(0, 1)$ in most cases. This quantity is close to 0.667 for most of the anomalies. A similar result has been obtained by Epstein and Zin (1991). The coefficient of risk aversion given by $1 - \lambda$ is estimated by $1 - \hat{\lambda}$. According to our estimation, this coefficient is close to 1 because $\hat{\lambda}$ is of order of 0.05 as in Epstein and Zin (1991). Moreover, we notice throughout our estimations that the inverse of the elasticity of intertemporal substitution does not coincide with the risk aversion supporting the idea of using a utility function that separates these two parameters.

[9] Net returns are computed using the transaction costs of Novy-Marx and Velikov (2016)

5.4 Comparison with the literature

Our analysis through Equation (22) can be compared to Equation (15) in He and Modest (1995) where they assume that the investor faces a proportional transaction costs in the investment process. He and Modest (1995) derive a lower and an upper bounds from the first order conditions. Since the proportional costs are assumed to be constant over time, these bounds are found to be constant. In our case, we do not derive these bounds but quantify the exact value of $E_t\left\{\beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)}R_{p,t+1}^{\frac{\lambda}{\rho}-1}R_{jt+1}\right\}$ as given in Equation (22). Interestingly, $E_t\left\{\beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)}R_{p,t+1}^{\frac{\lambda}{\rho}-1}R_{jt+1}\right\}$ is obtained to be time-varying and depends on several elements such as the time varying δ_t, the optimal selected portfolio at time t through ω_{it}, and the investor's preferences.

The parameter δ helps us to test for a given asset if trading costs have a significant effect on investors' behavior. In fact, a significant δ helps us through the relation (22) to see how inefficient will be the analytical solution of the portfolio selection problem obtained based on the relation (23). In such a situation ignoring trading costs could have disastrous consequences. Table 17 contains in Column 3 the estimates of the transaction costs by GMM using the formula $\hat{\delta}\hat{\beta}^{\hat{\lambda}/\hat{\rho}}$ (see Equation (22)) converted in percentage. We also report in the Column 2 the average trading costs on these strategies provided by Novy-Marx and Velikov (2016). We notice that our estimation for these strategies are quite close to the average trading costs obtained by Novy-Marx and Velikov (2016) using different estimation methods. Indeed, Novy-Marx and Velikov (2016) evaluate trading costs on anomalies using a Bayesian Gibbs Sampler on a generalized Roll (1984) model of stocks price dynamics. While we did our estimation based on a standard GMM procedure. In addition to the computation of trading costs, our estimation procedure allows us to test whether such costs have a significant effect on investors' actions in the financial market.

If δ_t is considered as a proxy for the transactions costs in the investment process, an interesting way to model this parameter could be $\delta_t = \delta_0 FL_t$ as in Farouh and Garcia (2020) where δ_0 is a positive constant to be estimated and FL_t is the funding liquidity at time t used as a measure of financial risk. To evaluate the effect of transaction costs in this situation, it suffices to estimate δ_0. In this case, Equation (12) becomes as follows

$$E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} \left(R_{t+1} - R^f \right) + \delta_0 F L_t \right\} = 0 \tag{24}$$

This new equation can then be used combined with Equation (9) to form the moment conditions to estimate δ_0 by GMM. The results of this analysis are reported in Tables 21-22 in Appendix. We obtain similar results as in Tables 4-5. Then, δ_t can be estimated as follows $\hat{\delta}_t = \hat{\delta}_0 F L_t$ which allows us to obtain the dynamic of the parameter δ_t over time.

6 Economic benefits from accounting for trading costs

In this section, we are going to measure the economic gain an investor can obtain when he accounts for trading costs in the portfolio selection process. This analysis will be done by comparing the out-of-sample performance of portfolio from our model (with trading costs) to the null model which ignores trading costs in the investment process. So, assumed that we have monthly data-set of size T_1. We also consider a finite life horizon (T_2 months with $T_2 < T_1$) investor who reallocates his portfolio at the end of each month of his life cycle. Then we use the first $T_1 - T_2$ information on the data-set to estimate unknown parameters about the vector of state variables in the optimization problem. Those estimations will help us to be able to implement the numerical procedure developed in Appendix A7 in order to obtain the portfolio rule at each period of time. Hence, at each time period of his life cycle ($t = T_1 - T_2 + 1, ..., T_1$), our investor finds portfolio weights to maximize the expected utility. The investor then holds those assets for a given period (a month), realizes gains and losses and recomputes optimal portfolio weights for the next period. This procedure is repeated for each time period through the investor's life cycle generating a time series of out-of-sample portfolio returns to evaluate the performance of the models. We compute look at the performance of two different strategies one which account for trading costs and the other one which ignores the trading costs. For this purpose, we need to assume a given form to the transaction costs in our model. A standard way to parametrize those frictions is to model them as proportional to the amount of rebalancing.

Let f_t denote the transaction cost per dollar of portfolio value. Then, we model f_t as follows:

$$f_t = \phi_p \mid y_t - \hat{y}_t \mid$$

where $\hat{y}_t$ is the proportion of the risky asset inherited from the previous period and given by

$$\hat{y}_t = \frac{y_{t-1}(1-k_{t-1})A_{t-1}(1-f_{t-1})R_t}{A_t} = \frac{y_{t-1}R_t}{y_{t-1}(R_t - R^f) + R^f}$$

where k_t is the fraction of the current income allocated to the consumption at time t and ϕ_p is the proportional cost parameter associated with the risky asset (see Lynch and Balduzzi (2000) for more details). A_t is the investor's income at time t defined according to the law of motion in Equation (3) with $R_{p,t+1}$ such that $R_{p,t+1} = (1-f_t)\left[y_t(R_{t+1} - R^f) + R^f\right]$ instead of (2) to account for the transaction cost. We still assume that the risky asset is one of the anomalies used in Novy-Marx and Velikov (2016) so that the parameter ϕ_p is given in Table 10 for each strategy. For example, when the risky asset in the economy is taken to be the industry-relative reversals (IRR), the proportional cost parameter ϕ_p is 1.78% with a significant trading cost effect according to our empirical results obtained in Section 5.

Several statistics such as the mean of the portfolio return (Mean), its standard deviation (SD), and the Sharpe Ratio (SR) will be used to evaluate the out-of-sample performance of our portfolio selection process. The SR is obtained using the following relation:

$$SR = \frac{E(Portfolio) - R_f}{\sigma_{Portfolio}}$$

Because $E(Portfolio)$ and $\sigma_{Portfolio}$ are unknown, we estimate those quantities by their empirical counterpart from the sample of the optimal portfolio returns.

We report these statistics in Tables 6 and 7 (for a 10 years horizon investor with $T = 120$) for two different anomalies. More importantly, we obtain Table 6 by using the parameter calibrated on the industry-relative reversals as the risky asset (ϕ_p=1.78%) and Table 7 with the parameter calibrated on the asset growth (ϕ_p=0.11%). The Panel A of those two tables gives statistics when accounting for trading costs in the portfolio selection problem and the panel B contains the same statistics when ignoring the trading costs. Moreover, we compute those statistics for two different values of the EIS (see Column 1 of each table) when the relative risk aversion is set to $\gamma = 6$. We report the out-of-sample mean of the optimal portfolio in Column 2, the out-of-sample volatility given by the standard deviation in Column 3 and the out-of-sample excess return per unit of deviation in Column 4.

Table 6 reveals that accounting for transaction costs permits to outperform the situation

Table 6: **Out-of-sample performance analysis for the Industry-relative reversals with** $\gamma = 6$

EIS	Mean	SD	SR
Panel A: Accounting for transaction costs			
0.8	0.16	0.0105	0.1349
2	0.09	0.0107	0.0715
Panel B: Ignoring transaction costs			
0.8	0.08	0.010	0.0487
2	0.03	0.0127	0.0231

where transaction costs are ignored in terms of the portfolio mean and the Sharpe ratio. For instance, we can see that the Sharpe ratio obtained when the $EIS = 0.8$ is 0.1349, about 2.76 times the Sharpe ratio when transaction costs are ignored. A similar result is obtained with $EIS = 2$. According to the SR there is a large economic gain from accounting for trading costs in the investment process when the risky asset is the IRR. This finding is consistent with the result of the empirical analysis about the effect of trading costs for this strategy. Indeed, we found empirically that the trading costs have a significant effect when the risky asset is assumed to the IRR. Thus, investors have to care about trading costs in such a situation in order to optimally behave in the financial market.

The second finding is that the effect of ignoring trading costs on the portfolio performance is more important for $EIS = 2$ than for $EIS = 0.8$. In fact, the effect of ignoring transaction costs is amplified by the fact that investors with $EIS > 1$ tend to be more aggressive on the financial market. More precisely, when $EIS < 1$, consumers' income effect is larger than their substitution effect so that investors prefer to consume more today and participate less to the financial market. In this situation, the effect of the transaction costs on the portfolio performance is attenuated by the fact that investors do not want to take risks in the financial market. However, $EIS > 1$ implies that the substitution effect is stronger than the income effect and investors prefer participation to the financial market in order to smooth the consumption in the future. This will amplify the effect of the trading costs on the the optimal portfolio performance.

When the risky asset in the economy is assumed to be the asset growth (Ag) (as in Table 7) the proportional cost parameter ϕ_p is 0.11%. We found through the empirical analysis

Table 7: **Out-of-sample performance analysis for the Asset growth with $\gamma = 6$**

EIS	Mean	SD	SR
Panel A: Accounting for transaction costs			
0.8	0.33	0.0110	0.3029
2	0.39	0.0114	0.3400
Panel B: Ignoring transaction costs			
0.8	0.31	0.0101	0.2974
2	0.38	0.0116	0.3254

that trading costs on this strategy do not have a significant effect on the investment decision according to the test procedure developed in Section 3.

We can notice from Table 7 that no significant difference in terms of out-of-sample performance exists between the two investment strategies. In fact, as we saw it in Section 5, trading costs have no effect on the investment decision for this asset. Thus, the two optimal investment policies are very close to each other.

The results of Tables 6 and 7 imply that if trading costs have no effect on investment decision according to our test procedure of Section 3, using trading costs in the portfolio selection process does not significantly improve the out-of-sample performance (see Table 7). In this context investors could ignore those frictions in their investment process to simplify their optimization problem. However, when a significant trading cost effect is obtained through the test procedure of Section 3, investors need to account for trading costs in the portfolio selection process in order to improve the out-of-sample performance of the optimal portfolio (see Table 6).

We also use an utility based statistic which is the certainty equivalent (CE) return. This is the most relevant metric to assess the out-of-sample performance since it quantifies benefits based on investors' preferences. Here, the CE represents the annualized risk-free return that gives the investor the same utility as the portfolio obtained without trading costs in the model. It is a form of compensation which makes the investor indifferent between the two investment strategies. When the CE > 0 investors ask a certain compensation to be added to the null model in order to obtain the same utility as in the model with trading costs. This implies that there is a gain from accounting for trading costs in the investment process. However, when the CE $\cong 0$, we conclude that there is no significant economic gain from accounting for trading costs in the portfolio selection process.

Table 8: **The Certainty Equivalent for two risky assets with** $\gamma = 6$

EIS	The industry relative reversals	The Asset Growth
0.8	0.0300	0.00367
2	0.0700	0.00524

Table 8 reports the CE for two risky assets across two different values of the EIS. We can notice through this table that the CE is very close to zero when the risky asset in the economy is assumed to be the Ag. Hence, accounting for transaction costs does not improve significantly the investor's utility compared to ignoring them. This result is due to the fact that trading costs have no effect on investment decision for this strategy as we saw it from our empirical results given in Tables 4 and 5. However, we obtain an important CE for the model with the IRR. According to this statistic, investors have to take into account trading costs in their investment process in order to improve the out-of-sample performance of the optimal portfolio in terms of the CE.

We also observe that the CE is larger for $EIS = 2$ compared to what we obtain for $EIS = 0.8$. This result means that investors with large EIS (for instance $EIS > 1$) ask for more compensation in order to be indifferent between the two strategies. Thus, as observed for the SR, the trading costs effect on the portfolio performance seems to be important for greater values of the EIS. This analysis about the trading cost effect on the portfolio performance also justifies the importance of distinguishing the relative risk aversion from the EIS.

7 Conclusion

In this paper we analyze a portfolio optimization problem of a recursive preference investor who faces transaction costs on stock market. In this context, we consider a simple economy with two assets including a risky asset and a risk-free asset.

We develop a simple test procedure based on a two-step GMM estimation which allows us to test whether transaction costs have a significant effect on investors welfare in the economy. An interesting property of this test procedure is that the results do not depend on the form of the trading costs assumed in the model. We also propose a two-step procedure to test overidentifying restrictions when one component of the parameter of interest could be at the boundary of its parameter space. We find through a simulation exercise that our two-step procedure has good properties for a wide choice of the nominal size of the first step of the procedure. Our procedure outperforms the J-test based on the adjusted critical value and the modified J-test proposed by Ketz (2019) when the nominal size of the first step is taken to be $\alpha_1 = 0.5\%$.

In an empirical analysis we apply our test procedures to the class of anomalies used in Novy-Marx and Velikov (2016). Not surprisingly, we find that trading costs have no effect when the risky asset is assumed to be the market portfolio. Nonetheless, trading costs have a significant effect in terms of utility costs for most of anomalies from Novy-Marx and Velikov (2016) in particular those whose trading costs exceed 1% of the gross return. Thus, it is important not to ignore such a friction when making investment decisions.

To measure the economic gain of accounting for transaction costs, we use a model with proportional trading costs and compare the out-of-sample performance in terms of the mean, SD, SR, and CE. We observe through this analysis that the investor significantly improves the out-of-sample performance of his portfolio only when a significant trading costs effect is detected by our test procedure of Section 3.

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Appendix A

Appendix A1: Details about the first order conditions of the consumption-investment optimization problem

Let λ_t and μ_t denote the Lagrange multiplier associated with the constraint in Equations (3) and (4) respectively. The Lagrangian associated with the maximization problem is

$$J_t\left(A_t, y_{t-1}, I_t\right) = \max_{C_t, A_{t+1}, y_t} \left\{ C_t + E_t\left[\frac{M_{t+1}}{M_t} J_{t+1}\left(A_{t+1}, y_t, I_{t+1}\right)\right] + \lambda_t\left(R_{p,t}A_t - T_t - C_t - A_{t+1}\right) - \mu_t\left(T_t - f(y_t)\right) \right\}$$

First order conditions: for every $t = 0, 1, ..., T-1$

$$\textit{with respect to } C_t: \quad 1 - \lambda_t = 0 \tag{25}$$

$$\textit{with respect to} A_{t+1}: \quad E_t\left[\frac{M_{t+1}}{M_t} J_1\left(t+1\right)\right] - \lambda_t = 0 \tag{26}$$

$$\textit{with respect to } y_t: \quad E_t\left[\frac{M_{t+1}}{M_t} J_2\left(t+1\right)\right] + \mu_t f'(y_t) = 0 \tag{27}$$

Envelope Conditions:

$$J_1(t) = \lambda_t R_{p,t} \tag{28}$$

$$J_2(t) = \lambda_t\left(R_t - R_f\right) A_t \tag{29}$$

Kuhn-tucker condition: for every $t = 0, 1, ..., T-1$

$$R_{p,t}A_t - T_t - C_t - A_{t+1} = 0 \tag{30}$$

$$\mu_t\left(T_t - f(y_t)\right) = 0 \tag{31}$$

$$\mu_t \geq 0 \tag{32}$$

The terminal conditions are: $A_{T+1} = T_T = 0$ and $J_T = C_T$.

By combining Equations (25), (26), and (28), we obtain

$$E_t\left[\frac{M_{t+1}}{M_t}R_{p,t+1}\right] = 1 \tag{33}$$

By combining Equations (25), (27), and (29), we get

$$E_t\left[\frac{M_{t+1}}{M_t}\left(R_{t+1} - R_f\right)A_{t+1}\right] + \mu_t f'(y_t) = 0. \tag{34}$$

Since $\mu_t \geq 0$, f is a non-decreasing function, and A_{t+1} is known at the time and positive (see Lemma 2 below) then:

$$E_t\left[\frac{M_{t+1}}{M_t}\left(R_{t+1} - R_f\right)\right] \leq 0 \tag{35}$$

where

$$\frac{M_{t+1}}{M_t} = \beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}. \tag{36}$$

With $\lambda = 1 - \gamma$. $\frac{M_{t+1}}{M_t}$ is called stochastic discount factor. To compute easily $\frac{M_{t+1}}{M_t}$ (the discount stochastic factor), one needs to recall that $R_{p,t+1} = \frac{J_{t+1}}{J_t - C_t}$, which is not an assumption but a definition based on Equations (5) and (33) (see details below).

Lemma 2 $A_{t+1} \geq 0$ for every $t = 0, 1, ..., T$

Proof of Lemma 2: Suppose that there exists a $t' \leq T$ such as $A_{t'} < 0$. So by the budget constraint, $C_{t'} + A_{t'+1} = R_{p,t'}A_{t'} - T_{t'} < 0$. Since by definition $C_{t'} \geq 0$ then $A_{t'+1} < 0$. This implies recursively that $A_{t'+2} < 0$, $A_{t'+3} < 0$,.....,$A_{T+1} < 0$. This contradicts the terminal condition $A_{T+1} = 0$.

Derivation of $\frac{M_{t+1}}{M_t} = \beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}$. Let us recall that $\frac{M_{t+1}}{M_t} = \frac{U_1(t+1)}{U_1(t)}U_2(t)$

$$\begin{aligned}
U_1(t) &= \frac{\partial U_t}{\partial C_t} = (1-\beta)C_t^{\rho-1}U_t^{1-\rho} \\
U_1(t+1) &= (1-\beta)C_{t+1}^{\rho-1}U_{t+1}^{1-\rho} \\
U_2(t) &= \frac{\partial U_t}{\partial U_{t+1}} = \frac{1}{\rho}\beta(\frac{\rho}{1-\gamma})\left[(1-\gamma)U_{t+1}^{-\gamma}\right]\left[E_t U_{t+1}^{1-\gamma}\right]^{\frac{\rho}{1-\gamma}-1}U_t^{1-\rho} \\
&= \beta U_{t+1}^{-\gamma}\left[E_t U_{t+1}^{1-\gamma}\right]^{\frac{\rho}{1-\gamma}-1}U_t^{1-\rho}.
\end{aligned}$$

Using these expressions we get:

$$\begin{aligned}\frac{M_{t+1}}{M_t} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} U_{t+1}^{1-\rho-\gamma}\left[E_t U_{t+1}^{1-\gamma}\right]^{\frac{\rho+\gamma-1}{1-\gamma}} \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}\left[\frac{U_{t+1}}{\eta(U_{t+1})}\right]^{1-\rho-\gamma}\end{aligned}$$

where $\eta(U_{t+1}) = \left[\mathbb{E}_t U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$. Now let us find $\left[\frac{U_{t+1}}{\eta(U_{t+1})}\right]$. Recall that by definition the return of the portfolio can be given by $R_{p.t+1} = \frac{J_{t+1}}{J_t - C_t}$ Where $J_t = \frac{U_t}{U_1(t)} = \frac{C_t}{1-\beta}\left(\frac{U_t}{C_t}\right)^{\rho}$. So $J_{t+1} = \frac{C_{t+1}}{1-\beta}\left(\frac{U_{t+1}}{C_{t+1}}\right)^{\rho}$ and $J_t - Ct = \frac{C_t}{1-\beta}\left[\left(\frac{U_t}{C_t}\right)^{\rho} - (1-\beta)\right]$. From the recursive utility function in Equation (1), we know that $U_t^{\rho} = (1-\beta)\, C_t^{\rho} + \beta\,[\eta(U_{t+1})]^{\rho}$.

This implies that $\left[\left(\frac{U_t}{C_t}\right)^{\rho} - (1-\beta)\right] = \beta\left[\frac{\eta(U_{t+1})}{C_t}\right]^{\rho}$. Then, $J_t - C_t = \beta\frac{C_t}{1-\beta}\left[\frac{\eta(U_{t+1})}{C_t}\right]^{\rho}$. With J_{t+1} and $J_t - C_t$ in hands, we can compute $R_{p.t+1}$:

$$\begin{aligned}R_{p.t+1} &= \frac{J_{t+1}}{J_t - C_t} \\ &= \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)\left[\frac{U_{t+1}}{\eta(U_{t+1})}\frac{C_t}{C_{t+1}}\right]^{\rho} \\ &= \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\rho}\left[\frac{U_{t+1}}{\eta(U_{t+1})}\right]^{\rho}\end{aligned}$$

which implies that $\left[\frac{U_{t+1}}{\eta(U_{t+1})}\right] = (\beta R_{p,t+1})^{\frac{1}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\rho-1}{\rho}}$. So $\frac{M_{t+1}}{M_t}$ can be rewritten as follows:

$$\begin{aligned}\frac{M_{t+1}}{M_t} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}\left(\beta R_{p,t+1}\right)^{\frac{1-\rho-\gamma}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\rho-\gamma)\frac{\rho-1}{\rho}} &(37)\\ &= \left(\beta\right)^{\frac{1-\gamma}{\rho}}\left(R_{p,t+1}\right)^{\frac{1-\gamma}{\rho}-1}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{\rho}(\rho-1)}. &(38)\end{aligned}$$

Appendix A2: Proof of Proposition 1

Proposition 1 is a corollary from Lemma 1 below.

Lemma 1. Assume that Assumption A holds and that θ_0 is such that $\delta = 0$ and $(\theta_2, \theta_3, \theta_4)$ are interior points of the parameter space. Then, the following results hold:

1. $\hat{\theta}$ is a consistent estimator of θ_0, i.e. $\hat{\theta} = \theta_0 + o_p(1)$.

2. $l_T(\theta)$ admits a quadratic expansion in θ given by:

$$l_T(\theta) = l_T(\theta_0) + \frac{1}{2}X_T'\mathcal{I}X_T - \frac{1}{2}q_T\left(\sqrt{T}\,(\theta-\theta_0)\right) + R_T(\theta)$$

where $\mathcal{I} = \Gamma' S^{-1}\Gamma$, $X_T = \mathcal{I}^{-1}\Gamma' S^{-1}\sqrt{T}G_T(\theta_0)$, $q_T(\lambda) = (\lambda - X_T)'\,\mathcal{I}\,(\lambda - X_T)$, $\lambda \in \mathbb{R}^4$ and for all $\gamma_T \to 0$, $\sup_{\theta\in\Theta,\|\theta-\theta_0\|\le\gamma_T}\left[\frac{|R_T(\theta)|}{(1+\|\sqrt{T}(\theta-\theta_0)\|)^2}\right] = o_p(1)$.

3. Let $\Lambda = \mathbb{R}^+ \times \mathbb{R}^3$. Let $\hat{\lambda}_T = \inf_{\lambda\in\Lambda} q_T(\lambda)$. Then, $\sqrt{T}(\hat{\theta}-\theta_0) = \hat{\lambda}_T + o_p(1)$.

4. Let $q_\delta(\lambda_\delta) = (\lambda_\delta - Z_\delta)^2 / \left(H\mathcal{I}^{-1}H'\right)$ where $Z_\delta \sim N\left(0, H\mathcal{I}^{-1}H'\right)$ and $q_\delta\left(\hat{\lambda}_\delta\right) = \inf_{\lambda_\delta\ge 0} q_\delta(\lambda_\delta)$. Then, $\sqrt{T}\hat{\delta} \xrightarrow{d} \hat{\lambda}_\delta$.

5. $\sqrt{T}\hat{\delta} \xrightarrow{d} \hat{\lambda}_\delta = Z_\delta I(Z_\delta \ge 0)$ so that $\hat{\delta}$ has a half-normal asymptotic distribution.

Proof of Lemma 1

1. Proof of consistency: As $g(Z_t,\theta)$ is continuous in θ and Θ is compact, the minimum $\hat{\theta}$ exists. Moreover, $\{g(Z_t,\theta_0)\}$ is continuous in Z_t and hence is stationary ergodic by Assumption A1. $G_T(\theta)$ satisfies a uniform law of large numbers by Assumption A4(i) (see for instance Hayashi (2000)). Therefore, $\hat{\theta}$ is a consistent estimator of θ_0.

2. We need to check the conditions GMM1*, GMM2, and GMM3 of Andrews (1997).

GMM1*:

GMM1*(a) requires that GMM1(a), GMM1(C), and GMM1(e) hold. GMM1(a) holds because $G_T(\theta)$ satisfies the law of large numbers, hence $G_T(\theta) \xrightarrow{P} G(\theta)$.

GMM1(c), namely $G(\theta_0) = 0$, is satisfied by Assumption A3.

GMM1(e) follows from the fact that $\widehat{S}$ does not depend on θ and $\widehat{S}$ is a consistent estimator of S by Assumption A5.

GMM1*(b) and (c) hold because the domain of $G(\theta)$ includes a set Θ^+ that satisfies conditions (i) and (ii) of Assumption 1*(a). Moreover, each element of the K vector valued function $G_T(\theta)$ has continuous right derivatives of order one on Θ^+ with probability 1.

GMM1*(d) holds because $\partial G_T(\theta)/\partial\theta$ converges in probability to $\partial G(\theta)/\partial\theta$ uniformly in θ on $\mathcal{N}$ by Assumption A4(ii).

GMM1*(e) holds because under Assumption A, $\partial G_T(\theta_0)/\partial\theta' \xrightarrow{P} \partial G(\theta_0)/\partial\theta' = \Gamma$.

GMM2:

Because $\{g(Z_t,\theta_0)\}$ is a martingale difference sequence (see Equations (9) and (12)) and the existence of S, we have a central limit theorem:

$$\sqrt{T}G_T(\theta_0) \xrightarrow{d} \mathcal{N}(0,S),$$

hence $\sqrt{T}G_T(\theta_0) = O_p(1)$ and GMM2 holds.

GMM3 is the same as our assumption A6.

By Theorem 7 of Andrews (1997), the expansion of $l_T(\theta)$ given in point 2 holds.

3. Point 3 follows from Theorem 3(a) of Andrews (1999). We need to check Assumptions 2 to 6 of Andrews (1999). By Theorem 7 of Andrews (1997), Assumptions GMM1, GMM2, and GMM3 imply Assumptions 1-3 of Andrews (1999). Assumption 4 (consistency) of Andrews (1999) follows from the point 1. Assumption 5 of Andrews (1999) holds with $B_T = b_T = \sqrt{T}$ and $\Lambda = \mathbb{R}^+ \times \mathbb{R}^3$. Assumption 6 of Andrews (1999) holds because the cone Λ is convex.

4. Point 4 follows from Theorem 4 and Corollary 1 of Andrews (1999).

5. Point 5 follows from the minimization of $q_\delta(\lambda)$.

Appendix A3: Proof of Proposition 2

If $\delta = 0$,

$$P\left[\{W > c_{\alpha_1}\} \cap \{J > c_{1\alpha_2}\} | H_0\right] \leq P\left[W > c_{\alpha_1} | H_0\right] \leq \alpha_1.$$

If $\delta > 0$,

$$P\left[\{W > c_{\alpha_1}\} \cap \{J > c_{1\alpha_2}\} | H_0\right] \leq P\left[J > c_{1\alpha_2} | H_0\right] \leq \alpha_2.$$

Hence, for $\delta \geq 0$, we have

$$P\left[\{W > c_{\alpha_1}\} \cap \{J > c_{1\alpha_2}\} | H_0\right] \leq \max(\alpha_1, \alpha_2).$$

Moreover, for $\delta \geq 0$,

$$P\left[\{W \leq c_{\alpha_1}\} \cap \{J > c_{2\alpha_3}\} | H_0\right] \leq P\left[J > c_{2\alpha_3} | H_0\right] \leq \alpha_3$$

because $c_{2\alpha_3} > c_{1\alpha_3}$ so that the J test based on $c_{2\alpha_3}$ has a size smaller than α_3 even when $\delta > 0$.

In summary, we obtain

$$\begin{aligned} P\left[\text{reject } H_0 | H_0\right] &\leq P\left[\{W > c_{\alpha_1}\} \cap \{J > c_{1\alpha_2}\} | H_0\right] \\ &\quad + P\left[\{W \leq c_{\alpha_1}\} \cap \{J > c_{2\alpha_3}\} | H_0\right] \\ &\leq \max(\alpha_1, \alpha_2) + \alpha_3. \end{aligned}$$

This completes the proof of Proposition 2.

Appendix A4: Algorithm to simulate the critical value of $0.5\chi^2(K-L)+0.5\chi^2(K-L+1)$

Let us denote by B the number of simulations. K is the number of moment conditions and L the number of estimated parameters.

- Initialize a vector J of size B: J=zeros(1,B),

- Then at each simulation step b=1,...,B generate a uniform random variable $x \in (0,1)$ (with Maltab $x = rand$)

- if $x > 0.5$ then generate J(b) from a $\chi^2(K-L+1)$ otherwise, J(b) is from a $\chi^2(K-L)$

- Repeat this process for b=1 to B.

- The critical value of $0.5\chi^2(K-L)+0.5\chi^2(K-L+1)$ at the significant level $\alpha\%$ is obtained using the $(100-\alpha)\%$ percentile of J (with Matlab $Jcritic = prctile(J, 100-\alpha)$).

Appendix A5: Calibrated parameters for simulations

The parameters used in the simulations are chosen to match the estimates obtained by least-squares using the actual time series from July 1973 to December 2013.

Market portfolio as anomaly

The marginal distribution of Y_t in this case is the following

$$\begin{pmatrix} X_t \\ r_{1,t} \\ r_{2,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} .0008754 \\ .0042422 \\ .0109538 \end{pmatrix}, \begin{bmatrix} .000015 & -2.4e-06 & .000033 \\ -2.4e-06 & .000015 & 6.2e-06 \\ .000033 & 6.2e-06 & .002319 \end{bmatrix}\right)$$

The other parameters are calibrated as follows

$$a = \begin{pmatrix} 0.0015 \\ 0 \\ 0.087 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0.009 \\ 0 & 0.823 & 0 \\ 0 & 7.83 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & 0.006 \\ 0 & 0.165 & 0.0031 \\ -1.180 & -7.597 & 0 \end{pmatrix}, Diag(\Sigma_\epsilon) = (0.852, 0.708, 0.861).$$

Seasonality as anomaly

The marginal distribution of Y_t in this case is the following

$$\begin{pmatrix} X_t \\ r_{1,t} \\ r_{2,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} .0008754 \\ .0042422 \\ .0072585 \end{pmatrix}, \begin{bmatrix} .000015 & -2.4e-06 & -9.4e-07 \\ -2.4e-06 & .000015 & 9.8e-06 \\ -9.4e-07 & 9.8e-06 & .001483 \end{bmatrix}\right)$$

The other parameters are calibrated as follows

$$a = \begin{pmatrix} 0.0015 \\ 0 \\ 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 & -0.436 & 0 \\ 0 & 0.8368 & 0 \\ 0 & 3.777 & 0.0908 \end{pmatrix}, \; C = \begin{pmatrix} 0.0911 & 0 & 0 \\ 0.01885 & 0.15216 & 0.00169 \\ 0 & 0 & 0.0928 \end{pmatrix}, \; Diag(\Sigma_\epsilon) =$$

$(0.862, 0.778, 0.890)$.

Appendix A6: Justification of Equation (22).

By multiplying (12) by y_t (the weight of the risky asset in the portfolio), we have that

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} y_t\left(R_{t+1}-R_f\right)\right]+\delta_t y_t=0 \tag{39}$$

and using the fact that $y_t\left(R_{t+1}-R_f\right)=R_{p,t+1}-R_f$, (39) becomes

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1}\left(R_{p,t+1}-R_f\right)\right]+\delta_t y_t=0 \tag{40}$$

which gives that

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}}\right]+\delta_t y_t=E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} R_f\right]. \tag{41}$$

By multiplying (41) by $\beta^{\frac{\lambda}{\rho}}$ and using (9) we obtain the following result

$$E_t\left[\beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} R_f\right]=1+\delta_t y_t \beta^{\frac{\lambda}{\rho}} \tag{42}$$

Moreover, by multiplying (12) by $1-y_t$ (the weight of the risk-free asset in the portfolio) and using the same technique as before, we obtain that

$$E_t\left[\beta^{\frac{\lambda}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\lambda}{\rho}(\rho-1)} R_{p,t+1}^{\frac{\lambda}{\rho}-1} R_{t+1}\right]=1-\delta_t\left(1-y_t\right) \beta^{\frac{\lambda}{\rho}}. \tag{43}$$

Therefore, (42) and (43) imply (22).

Appendix A7: The numerical procedure.

The results of Section 6 are obtained using the same numerical procedure as in Lynch and Balduzzi (1999) and Lynch and Balduzzi (2000). First, we need to discretize all the state variables in this optimization problem.

Discrete approximation of the set of state variables

Let us consider a simple case where the proportion of the portfolio in the risky asset y_t is in $[0,1]$ for $t = 1, ..., T$, we need to discretize this set into a grid of points. Thus, as in Lynch and Balduzzi (1999) and Lynch and Balduzzi (2000), the following grid of points on the interval [0,1] will be used to discetize y_t for all $t = 1, ..., T$: $y = \{0.00, 0.02, 0.04, ..., 0.96, 0.98, 1.00\}$ so that we obtain 50 discrete points for this variable.

Let $d_t = \log(1 + D_t)$, $r_t = \log(1 + R_t)$ with D_t the dividend yield and R_t the risky asset return. We assume that the vector of state variables $Q_t = (r_t, d_t)^{'}$ follows a VAR model:

$$Q_{t+1} = b + AQ_t + \epsilon_{t+1}$$

where $b = (b_1, b_2)^{'}$, $\epsilon_t = (e_{1t}, e_{2t})^{'} \sim$ iid $\mathcal{N}(0, \Sigma)$, and $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. We also assume that d_t is the only state variable in the VAR model that means : $a_{j1} = 0, j = 1, 2$. This last assumption about the investment opportunity set implies that the dividend yield is sufficient to well predict the risky-asset return (Fama and French (1988), Lynch and Balduzzi (1999) and Lynch and Balduzzi (2000)). Hence, the VAR model becomes:

$$\begin{cases} r_{t+1} = b_1 + a_{12} d_t + e_{1,t+1} \\ d_{t+1} = b_2 + a_{22} d_t + e_{2,t+1} \end{cases}$$

and this model will be estimated by OLS using data from U.S financial market.

But, since d_t depends on asset prices at the end of period t, the value of that regressor at the end of period $t + 1$ reflects changes in asset prices during $t + 1$ as does r_{t+1} so $E\left(e_{1,t+1} | d_{t+1}, d_t\right) \neq 0$ (see Stambaugh (1999)). Consequently, OLS estimators of coefficients of the first equation in the VAR model although consistent are biased and have sampling distributions that differ from those in the standard setting. Stambaugh (1999) shows that this bias is given by:

$$E\left(\hat{a}_{12} - a_{12}\right) = \frac{\sigma_{e_1 e_2}}{\sigma^2_{e_2}} E\left(\hat{a}_{22} - a_{22}\right)$$

It is a positive bias since the bias in $\hat{a}_{22}$ is negative and that the unexpected return $e_{1,t+1}$ is negatively correlated with the innovation in the dividend yield $e_{2,t+1}$. Empirically the value of $\frac{\sigma_{e_1 e_2}}{\sigma^2_{e_2}}$ is in the order of 10 to 20 so that the magnitude of the positive bias in $\hat{a}_{12}$ is many times the negative bias in $\hat{a}_{22}$. A bias-corrected OLS estimator has been proposed in the literature in particular by Stambaugh (1999) using a Bayesian approach. However, Lewellen (2004) shows that this correction can substantially understate, in some circumstances, dividend yield's predictive ability since this approach implicitly discards any information we have about $\hat{a}_{22} - a_{22}$. Hence, using the fact that the slope in a predictive regression is strongly correlated with the dividend yield's auto-correlation, Lewellen (2004) proposes the following bias-adjusted estimator:

$$\hat{a}_{12adj} = \hat{a}_{12} - \frac{\sigma_{e_1 e_2}}{\sigma^2_{e_2}} (\hat{a}_{22} - a_{22})$$

Because the dividend yield is a persistent variable, even if we do not know $\hat{a}_{22} - a_{22}$, a lower bound can be put on it using $a_{22} \approx 1$ which gives us an upper bound on the bias in $\hat{a}_{12}$.

Based on those estimations, the following procedure is used to have a discrete approximation for Q_t. First, the dividend yield is discretized as a first order autoregressive process (Tauchen and Hussey, 1991) to obtain a discrete process of nineteen points. For the return on the risky asset, we use the fact that the VAR model implies the following expression for the stock returns:

$$r_{t+1} = b_1 + a_{12} d_t + \nu e_{2,t+1} + u_{t+1}$$

where ν is the regression coefficient from regressing e_1 on e_2 and u_t is an i.i.d. normally distributed random variable with 0 mean and unknown variance σ^2_u, and assumed to be uncorrelated with e_2. The quadrature method is used to have a discrete distribution for u_t (with three points) calibrating σ^2_u by an estimator which is given by:

$$\hat{\sigma^2_u} = \frac{1}{T-1} \sum_{t=1}^{T} \hat{u_t}^2 = \frac{1}{T-1} \sum_{t=1}^{T} (e_{1t} - \hat{e_{1t}})^2$$

Then, we can have a discrete distribution for r_{t+1} for each $\{d_t, d_{t+1}\}$ since $e_{2,t+1} = d_{t+1} - b_2 - a_{22} d_t$, so $r_{t+1} = b_1 + a_{12} d_t + \nu (d_{t+1} - b_2 - a_{22} d_t) + u_{t+1}$. Hence, we obtain a discrete process for the asset return distribution with $19 \times 19 \times 3 = 1083$ which will be used to implement our numerical procedure. More details about this numerical method can be found in Lynch and Balduzzi (1999) and Lynch and Balduzzi (2000).

The estimation is done using data from the Federal Reserve Bank of St. Louis database. The VAR model gives us the following results in Table 9.

Table 9: **The VAR model estimation results**

Estimation results	$a_.$	$b_.$	Adjusted R^2
r_{t+1}	0.3191 (0.0719)	-0.0240 (0.0082)	0.025
d_{t+1}	0.9951 (0.0025)	3.88e-4 (2.87e-4)	0.9925

The result from regressing e_1 on e_2 provides $\hat{\nu} = -11.9831$ with a standard error of 0.9298 and the unknown variance σ^2_u is calibrated by $\hat{\sigma^2_u}$ so that we obtain 0.0064.

$\frac{\sigma_{e_1 e_2}}{\sigma^2_{e_2}}$ is calibrated to be -11.98 using data so that $\hat{a}_{12adj}$ is given by 0.2607.

We can now compute the investor's optimal investment strategy by solving the optimization problem for some values of the preference parameters and transaction costs parameter.

Appendix B

Table 10: The list of anomalies and their average transaction costs

Anomalies	Average trading costs (%)	Signal
Size	0.04	Market equity
Gross profitability	0.03	Gross profitability
Value	0.05	Book-to-market equity
ValProf combo	0.06	Sum of firms' ranks in univariate sorts on book-to-market and gross profitability
Accruals	0.09	Accruals
Asset growth	0.11	Asset growth
Investment	0.10	Investment
Piotroski's F-score	0.11	Piotroski's F-score
Net issuance	0.20	Net stock issuance
Return-on-book equity	0.38	Return-on-book equity
Failure probability	0.61	Failure probability
ValMomProf combo	0.43	Sum of firms' ranks in univariate sorts on book-to-market,gross profitability, and momentum
ValMom combo	0.41	Sum of firms' ranks in univariate sorts on book-to-market and momentum
Idiosyncratic volatility	0.52	Idiosyncratic volatility, measured as the residuals of regressions of their past three months' daily returns on the daily returns of the Fama-French three factors
Momentum	0.65	Prior year's stock performance excluding the most recent month
PEAD (SUE)	0.46	Standardized unexpected earnings (SUE)
PEAD (CAR3)	0.57	Cumulative three-day abnormal return around announcement (days minus one to one)
Industry momentum	1.22	Industry past month's return

Source: from Novy-Marx and Velikov (2016)

Table 11: List of anomalies and their average transaction costs (Continued)

Anomalies	Average trading costs (%)	Signal
Industry-relative reversals	1.78	Difference between a firm's prior month's return and the prior month's return of their industry
High-frequency combo	1.45	Sum of firms' ranks in the univariate sorts on industry relative reversals and industry momentum
Short-run reversals	1.65	Prior month's returns
Seasonality	1.46	Average return in the calendar month over the preceding five years
Industry-relative-reversals (Low volatility)	1.06	Industry relative reversals, restricted to stocks with idiosyncratic volatility lower than the NYSE median for the month

Source: from Novy-Marx and Velikov (2016)

Table 12: **Critical values for Proposition 1 with several significant levels**

Significant level (%)	0.5	1.5	2	2.5	3	3.5	4	4.5	6	6.5
Critical value	6.6262	4.709	4.223	3.844	3.545	3.292	3.068	2.878	2.416	2.293

Table 13: **GMM estimation result for testing trading costs effect**

Nondurable goods ($l = 3$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}$	J test	$\hat{\delta}$	J test
Market Portfolio	1.1202e-5	11.31	7.4085e-6	11.31
	(6.9561e-6)	(0.3339)	(3.0402e-6)	(0.3339)
Size	0.006213*	28.19+	0.0053058	28.2+
	(1.9759)	(0.001682)	(1.4745)	(0.001678)
Gross Profitability	0.0060357**	8.569	0.0054511	8.492
	(2.8466)	(0.5734)	(2.4264)	(0.5809)
Asset growth	0.00075158	9.813	2.7102e-11	10.19
	(0.054149)	(0.4571)	(6.6775e-17)	(0.4243)
Piotroski's F-score	0.006984*	19.43+	0.0047469	19.63+
	(2.6078)	(0.03516)	(1.2549)	(0.03292)
PEAD (SUE)	0.0050633*	8.428	4.8668e-14	10.26
	(2.6548)	(0.5871)	(2.6289e-22)	(0.4181)
Industry Momentum	0.015866***	12.3	8.9095e-12	13.13
	(6.1015)	(0.2653)	(2.3689e-18)	(0.2166)
Industry Relative Reversals	0.029084***	8.48	3.3939e-12	15.87
	(6.0485)	(0.5821)	(8.1042e-19)	(0.1033)
High Frequency Combo	0.0090193**	10.57	2.9413e-14	44.6+
	(4.6537)	(0.392)	(1.7572e-23)	(2.565e-16)
Short-run reversals	0.037915***	9.867	0.0042125	11.57
	(7.1833)	(0.4523)	(0.6927)	(0.3147)
Seasonality	0.023264***	13.58	4.4464e-13	15.84
	(10.7918)	(0.193)	(1.3648e-20)	(0.1043)
Industry Relative Reversals	0.0084407**	10.9	6.7388e-12	28.12+
(Low volatility)	(4.5955)	(0.3651)	(2.2958e-19)	(0.001724)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%

Table 14: **GMM estimation result for testing trading costs effect**

Nondurable goods and services ($l = 3$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}$	J test	$\hat{\delta}$	J test
Market Portfolio	1.5086e-11 (4.1184e-17)	13.03 (0.2219)	1.761e-12 (5.6067e-19)	13.03 (0.2218)
Size	0.003591* (2.1682)	20.89+ (0.02184)	0.0031195 (1.6788)	20.94+ (0.02153)
Gross Profitability	0.0035227* (2.2638)	7.147 (0.7115)	0.0032024 (2.2041)	7.96 (0.6327)
Asset growth	0.000987 (0.31822)	11.93 (0.29)	1.9722e-12 (1.2802e-18)	12.21 (0.2712)
Piotroski's F-score	0.0038143* (2.5751)	19.48+ (0.0346)	0.0025587 (1.2394)	19.6+ (0.03325)
PEAD (SUE)	0.0025297* (2.0475)	9.248 (0.5088)	1.5506e-12 (7.7305e-19)	11.45 (0.3238)
Industry Momentum	0.009534*** (5.6769)	10.04 (0.4369)	2.9285e-12 (8.9548e-19)	12.22 (0.2755)
Industry Relative Reversals	0.01484*** (5.7915)	7.521 (0.6756)	2.1991e-14 (8.4784e-23)	17.59 (0.0623)
High Frequency Combo	0.0043807** (3.9421)	9.754 (0.4623)	2.7008e-12 (4.1934e-19)	49.04 (4.003e-7)
Short-run reversals	0.019843*** (6.0628)	7.331 (0.6939)	0.0012486 (0.17565)	8.172 (0.06121)
Seasonality	0.01262*** (9.2385)	13.53 (0.1954)	9.8411e-13 (2.2143e-19)	19.34+ (0.03619)
Industry Relative Reversals (Low volatility)	0.0041517** (3.6257)	8.837 (0.5477)	1.9125e-12 (4.7762e-19)	28.15+ (0.001709)

* 10%, ** 5%, * * * 1%, + rejected at 5%

Table 15: **GMM estimation result for testing trading costs effect**

Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}$	J test	$\hat{\delta}$	J test
	Nondurable goods ($l = 1$)			
Market Portfolio	0.002353 (0.17381)	6.29^{+} (0.04306)	0.0023659 (0.16608)	5.85 (0.05366)
Size	0.0073 (1.543)	22.75^{+} (0.0002)	0.0063348 (1.1955)	22.74^{+} (1.153e-05)
Gross Profitability	0.0056 (1.280)	2.128 (0.345)	0.0050697 (1.1253)	2.155 (0.3405)
Asset growth	3.7017e-12 (6.712e-19)	3.805 (0.1492)	1.64e-13 (9.8206e-22)	5.283 (0.07125)
Piotroski's F-score	0.0047834 (1.1356)	13.14^{+} (0.001402)	0.0025528^{+} (0.32826)	13.34 (0.00127)
PEAD (SUE)	0.0033 (0.9505)	3.765 (0.1522)	2.2118e-14 (2.4805e-23)	6.916^{+} (0.03149)
Industry Momentum	0.0134^{**} (3.224)	4.563 (0.1021)	6.3617e-14 (5.2885e-23)	7.294^{+} (0.02607)
Industry Relative Reversals	0.0303^{*} (2.4073)	1.97 (0.3735)	3.0753e-13 (4.9125e-21)	6.493^{+} (0.03891)
High Frequency Combo	0.008^{**} (2.858)	4.342 (0.114)	1.0676e-12 (1.1622e-20)	41.16^{+} (1.154e-09)
Short-run reversals	0.041^{*} (2.299)	1.6 (0.4494)	0.0061406 (0.7371)	2.767 (0.2507)
Seasonality	0.025^{***} (8.525)	10.48^{+} (0.0053)	2.7857e-12 (4.8852e-19)	10.83^{+} (0.004442)
Industry Relative Reversals (Low volatility)	0.01^{*} (2.043)	3.284 (0.1936)	1.5166e-12 (6.6958e-20)	17.99^{+} (0.0001243)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%

Table 16: **GMM estimation result for testing trading costs effect**

Nondurable goods and services ($l = 1$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}$	J test	$\hat{\delta}$	J test
Market Portfolio	3.2804e-12 (1.6485e-18)	6.77^{+} (0.03387)	3.23e-12 (1.7249e-18)	7.657^{+} (0.02174)
Size	0.00273 (0.64431)	10.7^{+} (0.00474)	0.0022781 (0.63711)	14.59^{+} (0.0006783)
Gross Profitability	0.00344 (0.79065)	1.4 (0.4966)	0.0031298 (0.71032)	1.405 (0.4954)
Asset growth	0.0012 (0.3975)	5.379 (0.06791)	1.4792e-13 (3.4053e-21)	4.112 (0.128)
Piotroski's F-score	0.0035 (1.41)	7.395^{+} (0.02479)	0.0021802 (0.81194)	10.29^{+} (0.00583)
PEAD (SUE)	0.00256 (1.3346)	2.693 (0.2602)	9.2752e-13 (1.445e-19)	4.594 (0.1006)
Industry Momentum	0.00795^{**} (3.064)	4.42 (0.1097)	1.0147e-12 (4.3657e-20)	6.958^{+} (0.03084)
Industry Relative Reversals	0.0149^{*} (1.96)	2.367 (0.3061)	4.2774e-13 (1.7953e-20)	11.02^{+} (0.004038)
High Frequency Combo	0.00357^{*} (1.672)	3.406 (0.1822)	4.7894e-13 (6.0685e-21)	48.09^{+} (3.615e-11)
Short-run reversals	0.021^{*} (1.8231)	1.986 (0.3705)	0.0019814 (0.27817)	2.665 (0.2638)
Seasonality	0.0131^{***} (7.0885)	9.598^{+} (0.00824)	9.7158e-13 (1.7835e-19)	13.23^{+} (0.001341)
Industry Relative Reversals (Low volatility)	0.0041 (1.609)	3.572 (0.1676)	2.3293e-12 (4.2284e-19)	22.66^{+} (1.198e-05)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%

Table 17: **Comparison of trading costs**

Anomalies	Average trading costs	Our estimate trading costs
Size	0.04	0.27
Gross profitability	0.03	0.34
Asset growth	0.11	0.12
Piotroski's F-score	0.11	0.35
PEAD (SUE)	0.46	0.33
Industry momentum	1.12	1.34
Industry relative reversals	1.78	1.49
High-frequency combo	1.45	1.0
Short-run reversals	1.65	2.1
Seasonality	1.46	1.31
Industry Relative Reversals (Low volatility)	1.06	1.0

Table 18: **Estimates of the other parameters for each strategy with $l = 2$**

Estimates parameters	Nondurable goods and services		Nondurable goods	
	Ign TC	Net return	Ign TC	Net return
Market portfolio				
$\hat{\beta}$	0.99311	0.99310	0.99351	0.99340
	(0.00202)	(0.00204)	(0.00201)	(0.00203)
$\frac{\hat{\lambda}}{\rho}$	0.1083	0.1085	0.1398	0.1463
	(0.00679)	(0.00672)	(0.00667)	(0.0067)
$\hat{\rho}$	-0.4995	-0.49951	-0.3898	-0.3897
	(0.0857)	(0.0869)	(0.0774)	(0.0872)
Size				
$\hat{\beta}$	0.99315	0.99314	0.99342	0.99345
	(0.00227)	(0.00289)	(0.00286)	(0.00303)
$\frac{\hat{\lambda}}{\rho}$	0.107	0.1124	0.114	0.1126
	(0.00670)	(0.0210)	(0.00680)	(0.00780)
$\hat{\rho}$	-0.4989	-0.4998	-0.4709	-0.4802
	(0.07890)	(0.09802)	(0.0809)	(0.0889)
Gross profitability				
$\hat{\beta}$	0.9938	0.99371	0.99410	0.99370
	(0.00302)	(0.00368)	(0.00308)	(0.003452)
$\frac{\hat{\lambda}}{\rho}$	0.1129	0.1179	0.1273	0.1231
	(0.00689)	(0.02201)	(0.00709)	(0.00805)
$\hat{\rho}$	-0.4980	-0.49947	-0.4382	-0.43891
	(0.0840)	(0.0867)	(0.0871)	(0.0896)
Asset growth				
$\hat{\beta}$	0.9933	0.99310	0.99326	0.99329
	(0.00422)	(0.00454)	(0.00199)	(0.00256)
$\frac{\hat{\lambda}}{\rho}$	0.1181	0.1267	0.1051	0.1044
	(0.01432)	(0.02095)	(0.01521)	(0.0219)
$\hat{\rho}$	-0.4528	-0.4459	-0.5345	-0.5432
	(0.2715)	(0.2892)	(0.2184)	(0.2310)

Table 19: **Estimates of the other parameters for each strategy with $l = 2$**

Estimates parameters	Nondurable goods and services		Nondurable goods	
	Ign TC	Net return	Ign TC	Net return
Piotroski's F-score				
$\hat{\beta}$	0.99341 (0.00208)	0.99331 (0.00320)	0.9948 (0.00246)	0.99349 (0.00380)
$\frac{\hat{\lambda}}{\rho}$	0.110 (0.00778)	0.1131 (0.01978)	0.1097 (0.00728)	0.1107 (0.00805)
$\hat{\rho}$	-0.4983 (0.07482)	-0.49957 (0.0898)	-0.4897 (0.0871)	-0.4878 (0.0896)
PEAD (SUE)				
$\hat{\beta}$	0.99515 (0.00222)	0.99414 (0.00292)	0.9952 (0.00223)	0.99532 (0.00282)
$\frac{\hat{\lambda}}{\rho}$	0.1072 (0.00677)	0.109 (0.00978)	0.1112 (0.00671)	0.113 (0.00698)
$\hat{\rho}$	-0.4982 (0.0685)	-0.4978 (0.0968)	-0.4802 (0.0780)	-0.4780 (0.0835)
Industry Momentum				
$\hat{\beta}$	0.9918 (0.00318)	0.9921 (0.00379)	0.99310 (0.00313)	0.99324 (0.00339)
$\frac{\hat{\lambda}}{\rho}$	0.1147 (0.00705)	0.114 (0.02107)	0.1198 (0.00761)	0.1177 (0.00798)
$\hat{\rho}$	-0.4951 (0.07971)	-0.4972 (0.0848)	-0.4582 (0.0891)	-0.45890 (0.0901)
Industry Relative Reversals				
$\hat{\beta}$	0.99312 (0.00388)	0.99314 (0.00424)	0.9935 (0.002)	0.9934 (0.00235)
$\frac{\hat{\lambda}}{\rho}$	0.1086 (0.00678)	0.108 (0.00897)	0.141 (0.00657)	0.1392 (0.00852)
$\hat{\rho}$	-0.4992 (0.0834)	-0.4989 (0.0857)	-0.3858 (0.0745)	-0.3921 (0.0849)

Table 20: **Estimates of the other parameters for each strategy with $l = 2$**

Estimates parameters	Nondurable goods and services		Nondurable goods	
	Ign TC	Net return	Ign TC	Net return
High Frequency Combo				
$\hat{\beta}$	0.99348 (0.00227)	0.99370 (0.00321)	0.9939 (0.00249)	0.99387 (0.00367)
$\frac{\hat{\lambda}}{\rho}$	0.1117 (0.00688)	0.1092 (0.01692)	0.112 (0.00729)	0.112 (0.00802)
$\hat{\rho}$	-0.4987 (0.07581)	-0.49901 (0.0859)	-0.4898 (0.0867)	-0.4887 (0.0898)
Short-run reversals				
$\hat{\beta}$	0.99512 (0.00227)	0.99417 (0.00298)	0.9958 (0.00229)	0.99528 (0.00289)
$\frac{\hat{\lambda}}{\rho}$	0.1074 (0.00787)	0.1098 (0.00987)	0.1119 (0.00672)	0.1104 (0.00796)
$\hat{\rho}$	-0.4980 (0.0682)	-0.4974 (0.0954)	-0.48131 (0.0765)	-0.4820 (0.0842)
Seasonality				
$\hat{\beta}$	0.9938 (0.00328)	0.9942 (0.00389)	0.99341 (0.00312)	0.99328 (0.00362)
$\frac{\hat{\lambda}}{\rho}$	0.1142 (0.00697)	0.1145 (0.01907)	0.118 (0.00762)	0.1183 (0.00794)
$\hat{\rho}$	-0.4957 (0.07912)	-0.4978 (0.0885)	-0.4589 (0.0898)	-0.4579 (0.0900)
Industry Relative Reversals (Low volatility)				
$\hat{\beta}$	0.99317 (0.00488)	0.99416 (0.00554)	0.9932 (0.0021)	0.99345 (0.00247)
$\frac{\hat{\lambda}}{\rho}$	0.1105 (0.00670)	0.110 (0.00798)	0.1126 (0.00754)	0.1114 (0.00799)
$\hat{\rho}$	-0.4970 (0.07702)	-0.4980 (0.0807)	-0.4858 (0.0768)	-0.4921 (0.0907)

Table 21: **GMM estimation using funding liquidity**

Strategy	Ignoring transaction costs		Using the net return	
Nondurable goods ($l = 2$)				
	$\hat{\hat{\delta}}_0$	J test	$\hat{\hat{\delta}}_0$	J test
Market Portfolio	1.372e-05	9.017	1.0621e-05	9.061
	(9.8241e-06)	(0.1726)	(5.367e-06)	(0.1702)
Size	0.0074014*	24.79^{+}	0.004486	26.91^{+}
	(1.6671)	(0.00037)	(1.3289)	(0.00015)
Gross Profitability	0.0031331*	4.698	0.00258	5.421
	(1.724)	(0.5831)	(1.6102)	(0.4911)
Asset growth	3.2072e-13	7.582	5.227e-13	8.682
	(9.437e-21)	(0.2704)	(2.98e-20)	(0.1923)
Piotroski's F-score	0.002771*	17.00^{+}	0.0022506	17.40^{+}
	(2.201)	(0.0093)	(0.8891)	(0.0079)
PEAD (SUE)	0.003781*	7.181	6.7080e-13	9.510
	(1.6851)	(0.3040)	(3.8726e-20)	(0.1469)
Industry Momentum	0.009378**	6.810	1.241e-14	10.34
	(4.489)	(0.3388)	(4.7597e-24)	(0.1110)
Industry Relative Reversals	0.03072**	5.27	3.697e-13	1.83
	(5.413)	(0.5097)	(1.2829e-21)	(0.9347)
High Frequency Combo	0.005841**	5.341	4.9634e-12	42.6^{+}
	(3.079)	(0.5009)	(3.789e-19)	(1.3995e-07)
Short-run reversals	0.02724***	5.792	0.003541	6.293
	(5.9210)	(0.4469)	(0.6798)	(0.3912)
Seasonality	0.01921***	12.40	5.2047e-12	13.29^{+}
	(9.097)	(0.0536)	(2.679e-18)	(0.0387)
Industry Relative Reversals	0.003892**	3.280	8.1898e-13	23.20^{+}
(Low volatility)	(3.7956)	(0.7730)	(3.7841e-20)	(7.3219e-04)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%

Table 22: **GMM estimation using funding liquidity (continued)**

Nondurable goods and services ($l = 2$)				
Strategy	***Ignoring transaction costs***		***Using the net return***	
	$\hat{\delta}_0$	J test	$\hat{\delta}_0$	J test
Market Portfolio	1.009e-12	11.21	1.2e-12	12.81^{+}
	(1.942e-19)	(0.0821)	(6.5e-19)	(0.0462)
Size	0.0018094	16.56^{+}	0.00327	17.31^{+}
	(1.2071)	(0.0110)	(0.8942)	(0.0082)
Gross Profitability	0.00081	6.48	0.00121	2.92
	(1.573)	(0.3716)	(1.0234)	(0.8188)
Asset growth	0.00147	8.12	5.028e-13	10.287
	(0.2921)	(0.2294)	(2.293e-19)	(0.1131)
Piotroski's F-score	0.00140^{*}	17.43^{+}	0.001251	15.12^{+}
	(2.1990)	(0.0078)	(1.142)	(0.0193)
PEAD (SUE)	0.002021^{*}	7.342	1.0126e-14	7.321
	(1.9012)	(0.2904)	(4.783e-23)	(0.2922)
Industry Momentum	0.01042^{***}	9.38	1.097e-12	14.29^{+}
	(7.039)	(0.1533)	(5.549e-18)	(0.0266)
Industry Relative Reversals	0.002121^{***}	6.29	2.29e-13	15.35^{+}
	(6.3601)	(0.3915)	(5.981e-17)	(0.0177)
High Frequency Combo	0.00372^{*}	8.531	5.221e-12	27.82^{+}
	(2.1901)	(0.2017)	(3.975e-18)	(1.0159e-04)
Short-run reversals	0.010218^{*}	4.321	0.002821	7.741
	(1.981)	(0.6333)	(0.9107)	(0.2577)
Seasonality	0.0027^{***}	10.02	2.1329e-12	11.31
	(7.6821)	(0.1238)	(6.562e-18)	(0.0793)
Industry Relative Reversals	0.00461^{**}	6.821	1.12e-13	20.12^{+}
(Low volatility)	(3.812)	(0.3377)	(6.237e-20)	(0.0026)

$*$ 10%, $**$ 5%, $***$ 1%, + rejected at 5%