Cole, Stephen, Jonathan R. Cole, and Gary A. Simon. 1981. Chance and Consensus in Peer Review. *Science* 214 (November), 881–886.

Debreu, Gerard. 1991. Economic Theory in the Mathematical Mode. *American Economic Review* 74 (3): 267–278.

Dennett, Daniel. 1987. *The Intentional Stance.* Cambridge, MA: MIT Press.

Intriligator, Michael. 1971. *Mathematical Optimization and Economic Theory.* Englewood Cliffs, NJ: Prentice-Hall.

Kuhn, H. W., and A. W. Tucker. 1951. Nonlinear Programming. In *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,* ed. J. Neyman. Berkeley: University of California Press.

Kuhn, Thomas. 1962. *The Structure of Scientific Revolutions.* Chicago: University of Chicago Press.

Samuelson, Paul. 1947. *Foundations of Economic Analysis.* Cambridge, MA: Harvard University Press.

Simon, Herbert. 1955. A Behavioral Model of Rational Choice. *Quarterly Journal of Economics* 69 (1): 99–118.

Gregory A. Lilly

METHOD OF MOMENTS

Both the method of moments and its generalization, namely the generalized method of moments (GMM), have taken a prominent place in statistical inference in the social sciences and have been applied in almost every field of economics, including asset pricing, business cycle, commodity market, education, and labor market. The (standard) method of moments consists of estimating a parameter β by equating sample moments with population moments and solving these equations for β. This method was introduced at the end of the nineteenth century by the British mathematician Karl Pearson and was partly abandoned when the British biologist Ronald Fisher (1890–1962) showed that the maximum likelihood estimator (MLE) is more efficient than the method of moments estimator because its variance is smaller. Since its introduction by Lars Peter Hansen in 1982, however, GMM has been extremely popular in economics for three reasons. The first reason is that economic models are often too complex to be completely specified and an attempt to describe the full model is likely to yield misspecification errors. GMM provides a way to estimate partially specified models. The second reason is that, even if the model is completely specified, MLE may be too cumbersome to implement, whereas GMM provides a practical method to perform inference. Finally, GMM embeds many familiar estimation techniques as special cases, including the method of moments, ordinary least squares, instrumental variables estimation, and even maximum likelihood estimation.

THE ESTIMATION METHOD

GMM may be illustrated in a time series context where observations $x_1, x_2, \ldots, x_T$ are available. Assume that for a given function g, the moment condition $E[g(x_t,\beta)] = 0$ holds for a unique solution β. To solve this equation, the population mean needs to be replaced by the sample mean $g_T(\beta) = \frac{1}{T}\sum_{t=1}^{T} g(x_t, \beta)$. Two cases are distinguished, depending on whether the dimension of g is the same as or larger than the dimension of β. In the first case (just identified), the equation $g_T(\beta) = 0$ can be solved to obtain the method of moments estimator of β. In the second case (overidentified), the previous equation does not have a solution and the standard method of moments needs to be modified. The GMM estimator is defined as the solution $b_T = \underset{\beta}{\operatorname{argmin}}\; g_T(\beta)' W_T g_T(\beta)$ where W_T is a positive definite matrix that attaches weights to moments. For any W_T, b_T is consistent, that is, it approaches the true value β as the sample size T grows. Additionally, the GMM estimator has minimal variance if W_T is an estimator of the inverse of the variance of $g_T(\beta)$.

In the overidentified case, $g_T(b_T)$ is not exactly equal to zero but should go to zero as T goes to infinity. This provides the basis for the overidentifying restrictions test. This test consists of rejecting the hypothesis that the moment conditions hold in the population if $Tg_T(b_T)' W_T g_T(b_T)$ is greater than the critical value given by a chi-square distribution with degrees of freedom equal to the difference in the dimensions of g and β.

GMM provides a framework that encompasses most estimation techniques used in economics. Instrumental variables estimation, although a predecessor to GMM, can be recast as a special case of GMM. Consider the regression

$$y_t = x_t'\beta + \varepsilon_t$$

where x_t is endogenous, that is, correlated with the residual ε_t. As a consequence of endogeneity, the ordinary least squares estimator is not consistent. A consistent estimator, however, may be obtained by using a vector of so-called instruments z_t. To be a valid instrument, z_t must be correlated with x_t but not with the error ε_t. Then, β can be estimated by GMM using $g(y_t, x_t, z_t, \beta = (y_t - x_t'\beta)z_t$.

The resulting estimator is called the instrumental variables estimator.

MLE itself can be interpreted as a GMM estimator because the expectation of the derivative of the log-likelihood is equal to zero, giving rise to a moment condition.

To overcome computational difficulties, Daniel McFadden (1989) and Ariel Pakes and David Pollard (1989) have proposed the method of simulated moments

(MSM), which consists of replacing population moments with moments computed from simulated data.

MIXED POPULATION EXAMPLE

Consider a population where each subject is equally likely to be a male or a female. We observe a measure x (for instance, the weight) but not the sex and we wish to estimate the difference between males and females. This is particularly relevant in anthropology where most fossil specimens lack indicators of sex. Assume that observations are normally independently distributed with mean μ_F and variance σ^2 if the subject is a female and with mean μ_M and the same variance σ^2 if the subject is a male. The method of moments that matches the expectations of x,x^2, and x^4 with their sample counterparts permits to estimate μ_F, μ_M, and σ^2.

CONSUMPTION-BASED ASSET PRICING MODEL

Lars Peter Hansen and Kenneth Singleton (1982) explain how to apply GMM to estimate behavioral parameters of economic agents in a general equilibrium model, without having to describe the full economic environment. This approach may be used to study how agents allocate their spending. Consider an economy where a representative agent chooses consumption and investment plans so as to maximize $E\left[\sum_{i=0}^{\infty}\delta^i U\left(c_{t+i}\right) \mid \Omega_t\right]$, where c_t is consumption in period t, U is a utility function, δ is a discount factor, and Ω_t is the information at t. Maximization is performed under a budget constraint $c_t + p_t q_t = r_t q_{t-1} + w_t$

where w_t is the income, q_t the quantity of asset held at the end of period t, p_t the price of this asset, and r_t its return. The first order condition is $E\left[\delta \frac{r_{t+1}}{p_t}\frac{U'\left(c_{t+1}\right)}{U'\left(c_t\right)} - 1 \mid \Omega_t\right] = 0.$

Assume that $U(c_t) = (c_t^r - 1)/\gamma$. The parameter of interest $\beta = (\delta,\gamma)$ may be estimated by GMM using

$$g\left(r_{t+1}, p_t, c_{t+1}, c_t, z_t, \beta\right) = \left(\delta \frac{r_{t+1}}{p_t}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma-1} - 1\right) z_t$$

where z_t is a vector of instruments. Any element of Ω_t may be used as instrument, for example the constant and

$$\frac{r_{t+1-j}}{p_{t-j}}, \frac{c_{t+1-j}}{c_{t-j}}, j = 1, 2, \ldots, L.$$

DISCRETE CHOICE MODEL

Consider a model where each individual has the choice among J alternatives (for example, occupations or means of transportation). The individual chooses the alternative with the greatest value. The value u_{ij} of occupation j for individual i depends on a set of observed variables x_{ij} (such as sex, race, age, and education) so that $u_{ij} = x_{ij}\beta^j + \varepsilon_{ij}$ where $\varepsilon_j = (\varepsilon_{1j}, \varepsilon_{2j}, \ldots, \varepsilon_{nj})$ is normally distributed with mean zero and covariance matrix Σ. We observe that individual i chooses alternative j, if $u_{ij} \geq u_{il}$ for $l = 1, \ldots, J$. The probability of choosing alternative j, denoted by P_{ij}, involves a $J-1$ dimensional integral. If J is large, this integral is cumbersome to compute and hence maximum likelihood estimation is intractable. By contrast, P_{ij} can be estimated using simulations. Let ε_j^γ, $r = 1, \ldots, R$ be independently drawn from a normal distribution with mean zero and covariance Σ, and let $u_{ij}^\gamma = x_{ij}\beta^j + \varepsilon_{ij}^\gamma$. Then, an estimate $\hat{P}_{ij}$ of P_{ij} is given by the proportion of cases where u_{ij}^γ exceeds u_{il}^γ for all l different from j. An MSM estimator of $(\beta^1, \beta^2, \ldots, \beta^J$, is given by the solution to $\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{J} z_{ij}\left(y_{ij} - \hat{P}_{ij}\right) = 0$ where z_{ij} is exogenous (equal to x_{ij}, for instance).

SEE ALSO *Inference, Statistical; Instrumental Variables Regression; Large Sample Properties; Pearson, Karl*

BIBLIOGRAPHY

Hansen, Lars Peter. 1982. Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica* 50 (4): 1029–1054.

Hansen, Lars Peter, and Kenneth J. Singleton. 1982. Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models. *Econometrica* 50 (5): 1269–1286.

McFadden, Daniel L. 1989. A Method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration. *Econometrica* 57(5): 995–1026.

Pakes, Ariel, and David Pollard. 1989. Simulation and the Asymptotics of Optimization Estimators. *Econometrica* 57 (5): 1027–1057.

Marine Carrasco

MICROELECTRONICS INDUSTRY

The global Internet economy had its origins in the microelectronics industry and the innovation of the microchip in its various versions. In the stages of optoelectronics, the use of electrical energy to generate optical energy or vice versa such as light-emitting diodes (LEDs), the microelectronic chip, as developed by Intel Corporation, assumed enormous importance for the world of information and telecommunications.