Method of Moments

GMM may be illustrated in a time series context where observations $x_1, x_2, \ldots, x_T$ are available. Assume that for a given function $g$, the moment condition $E[g(x_t, \beta)] = 0$ holds for a unique solution $\beta$. To solve this equation, the population mean needs to be replaced by the sample mean $g_T(\beta) = \frac{1}{T} \sum_{i=1}^{T} g(x_t, \beta)$. Two cases are distinguished, depending on whether the dimension of $g$ is the same as or larger than the dimension of $\beta$. In the first case (just identified), the equation $g_T(\beta) = 0$ can be solved to obtain the method of moments estimator of $\beta$. In the second case (overidentified), the previous equation does not have a solution and the standard method of moments needs to be modified. The GMM estimator is defined as the solution $b_\tau = \arg\min_{\beta} g_T(\beta) W_T g_T(\beta)$ where $W_\tau$ is a positive definite matrix that attaches weights to moments. For any $W_\tau$, $b_\tau$ is consistent, that is, it approaches the true value $\beta$ as the sample size $T$ grows. Additionally, the GMM estimator has minimal variance if $W_\tau$ is an estimator of the inverse of the variance of $g_T(\beta)$.

In the overidentified case, $g_T(\beta)$ is not exactly equal to zero but should go to zero as $T$ goes to infinity. This provides the basis for the overidentifying restrictions test. This test consists of rejecting the hypothesis that the moment conditions hold in the population if $T g_T(\beta) W_T g_T(\beta)$ is greater than the critical value given by a chi-square distribution with degrees of freedom equal to the difference in the dimensions of $g$ and $\beta$.

GMM provides a framework that encompasses most estimation techniques used in economics. Instrumental variables estimation, although a predecessor to GMM, can be recast as a special case of GMM. Consider the regression

$$y_t = x_t' \beta + \varepsilon_t$$

where $x_t$ is endogenous, that is, correlated with the residual $\varepsilon_t$. As a consequence of endogeneity, the ordinary least squares estimator is not consistent. A consistent estimator, however, may be obtained by using a vector of so-called instruments $z_t$. To be a valid instrument, $z_t$ must be correlated with $x_t$ but not with the error $\varepsilon_t$. Then, $\beta$ can be estimated by GMM using $g(y_t, x_t, z_t, \beta) = (y_t - x_t' \beta) z_t$

The resulting estimator is called the instrumental variables estimator.

MLE itself can be interpreted as a GMM estimator because the expectation of the derivative of the log-likelihood is equal to zero, giving rise to a moment condition.

To overcome computational difficulties, Daniel McFadden (1989) and Ariel Pakes and David Pollard (1989) have proposed the method of simulated moments.
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(MSM), which consists of replacing population moments with moments computed from simulated data.

MIXED POPULATION EXAMPLE
Consider a population where each subject is equally likely to be a male or a female. We observe a measure \(x\) (for instance, the weight) but not the sex and we wish to estimate the difference between males and females. This is particularly relevant in anthropology where most fossil specimens lack indicators of sex. Assume that observations are normally independently distributed with mean \(\mu_x\) and variance \(\sigma^2\) if the subject is a female and with mean \(\mu_M\) and the same variance \(\sigma^2\) if the subject is a male. The method of moments that matches the expectations of \(x, x^2\), and \(x^3\) with their sample counterparts permits to estimate \(\mu_F, \mu_M\) and \(\sigma^2\).

CONSUMPTION-BASED ASSET PRICING MODEL
Lars Peter Hansen and Kenneth Singleton (1982) explain how to apply GMM to estimate behavioral parameters of economic agents in a general equilibrium model, without having to describe the full economic environment. This approach may be used to study how agents allocate their spending. Consider an economy where a representative agent chooses consumption and investment plans so as to maximize
\[
EU_c(t) = \gamma \delta^{\tau} U(c(t) - 1) + \gamma^{t+1} U(c(t+1) - 1)|\Omega_t|
\]
in period \(t\), \(U\) is a utility function, \(\delta\) is a discount factor, and \(\Omega_t\) is the information at \(t\). Maximization is performed under a budget constraint
\[
w_t + p_t f_t = r f_{t-1} + w_{t-1}
\]
where \(w_t\) is the income, \(f_t\) the quantity of asset held at the end of period \(t\), \(p_t\) the price of this asset, and \(r_t\) its return. The first order condition is
\[
E \left[ \delta^{\tau} \frac{U'(c(t))}{p_t} U'(c(t+1)) - 1 | \Omega_t \right] = 0.
\]

Assume that \(U'(c) = (c^{\gamma} - 1)^{\gamma}/\gamma\). The parameter of interest \(\beta = (\delta, \gamma)\) may be estimated by GMM using
\[
g(\tau_{t+1}) = \left[ \delta^{\tau_{t+1}} \frac{U'(c_{t+1})}{p_t} U'(c_t) - 1 \right] z_t
\]
where \(z_t\) is a vector of instruments. Any element of \(\Omega_t\) may be used as instrument, for example the constant and
\[
\frac{\tau_{t+1}}{p_{t+1}}, \frac{\tau_{t+1}}{p_{t+1}}, j = 1, 2, \ldots, L.
\]

DISCRETE CHOICE MODEL
Consider a model where each individual has the choice among \(J\) alternatives (for example, occupations or means of transportation). The individual chooses the alternative with the greatest value. The value \(u_{ij}\) of occupation \(j\) for individual \(i\) depends on a set of observed variables \(x_{ij}\) (such as sex, race, age, and education) so that
\[
u_{ij} = x_{ij}' \beta + \epsilon_{ij}
\]
where \(\epsilon_{ij} = (\epsilon_{ij1}, \epsilon_{ij2}, \ldots, \epsilon_{ijJ})\) is normally distributed with mean zero and covariance matrix \(\Sigma\). We observe that individual \(i\) chooses alternative \(j\) if \(u_{ij} \geq u_{il}\) for \(l = 1, \ldots, J\). The probability of choosing alternative \(j\), denoted by \(P_{ij}\), involves a \(J-1\) dimensional integral. If \(J\) is large, this integral is cumbersome to compute and hence maximum likelihood estimation is intractable. By contrast, \(P_{ij}\) can be estimated using simulations. Let \(\epsilon_{ijr}^{\prime}, r = 1, \ldots, R\) be independently drawn from a normal distribution with mean zero and covariance \(\Sigma\), and let \(u_{ij}^* = x_{ij}' \beta + \epsilon_{ijr}^{\prime}\). Then, an estimate \(\hat{P}_{ij}\) of \(P_{ij}\) is given by the proportion of cases where \(u_{ij}^*\) exceeds \(u_{il}^*\) for all \(l \neq j\) different from \(j\). An MSM estimator of \((\beta', \Sigma')\) is given by the solution to
\[
\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^J z_{ij} (y_{ij} - \hat{P}_{ij}) = 0
\]
where \(z_{ij}\) is exogenous (equal to \(x_{ij}\) for instance).

SEE ALSO Inference, Statistical; Instrumental Variables Regression; Large Sample Properties; Pearson, Karl

BIBLIOGRAPHY

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MICROELECTRONICS INDUSTRY
The global Internet economy had its origins in the microelectronics industry and the innovation of the microchip in its various versions. In the stages of optoelectronics, the use of electrical energy to generate optical energy or vice versa such as light-emitting diodes (LEDs), the microelectronic chip, as developed by Intel Corporation, assumed enormous importance for the world of information and telecommunications.