Parliament Shapes and Sizes

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Abstract

This paper proposes a model of Parliamentary institutions in which a Parliament Designer makes three decisions: whether a Parliament should comprise one or two chambers, what the relative bargaining power of each chamber should be if the Parliament is bicameral, and how many legislators should sit in each chamber. We document empirical regularities across countries that are consistent with the predictions of our model.

1 Introduction

Parliamentary institutions vary widely across countries. For instance, the Indian Lower Chamber – Lok Sabha – has 543 to 545 seats, and the Indian Upper Chamber – Rajya Sabha – has 250. The US Congress has 535 seats: 435 in the House of Representatives and 100 in the Senate. The Luxembourg Parliament has 60 legislators, in a single chamber.[1] As of 2012, there were 58

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[1] Throughout the paper, we use the term "legislator" to refer to a member of Parliament. If a Parliament is unicameral, we refer to its chamber as the House or the Lower Chamber indifferently. If there is a second chamber, we refer to it as the Upper Chamber or Senate indifferently.

bicameral and 110 unicameral systems recorded in the Database of Political Institutions (Beck et al. 2001 [updated in 2012]).

The main questions that arise when designing Parliamentary institutions are fundamentally quantitative: should there be one or two chambers, what should be their respective bargaining power, what should be the size of Parliament, etc. James Madison in the *Federalist No. 10* postulates a concave and increasing relationship between the population and the number of representatives.[2] In *Federalist No. 62*, he cites the large size of the Lower Chamber to argue in favor of an Upper Chamber.[3]

However, little is known on the institutional regularities of Parliaments across countries. Indeed, only two stylized facts have been documented: a log-linear relationship between the size of population and the size of Parliament (Stigler 1976), and an increasing relationship between the population size and the probability to have a bicameral Parliament (Massicotte 2001).

The purpose of this paper is twofold: to propose a model of Parliament design and to document empirical institutional regularities across countries, following the lead of the predictions of the model. One of our contributions is that our simple model generates predictions concerning variables that have an unambiguous equivalent in the data we observe.

In the model, a Parliament Designer makes three decisions: whether a Parliament should comprise one or two chambers, what the relative bargaining power of each chamber should be if the Parliament is bicameral, and how many legislators should sit in each chamber.

The Designer's aim is to maximize the expected utility of the representative agent of a discrete population of individuals. Each individual's utility is the opposite of the quadratic difference between the policy adopted by the Parliament and the individual's bliss point, which are both real numbers, minus the costs associated with the functioning of Parliament.

[2] "In the first place, it is to be remarked that, however small the republic may be, the representatives must be raised to a certain number, in order to guard against the cabals of a few; and that, however large it may be, they must be limited to a certain number, in order to guard against the confusion of a multitude. Hence, the number of representatives in the two cases [will be] proportionally greater in the small republic." (Madison 1787)

[3] "The necessity of a senate is not less indicated by the propensity of all single and numerous assemblies, to yield to the impulse of sudden and violent passions, and to be seduced by factious leaders into intemperate and pernicious resolutions." (Madison 1788)

We assume that the population is partitioned into parties and that two types of issues may arise: partisan and non-partisan. For non-partisan issues, individuals' bliss points are i.i.d., whereas for partisan issues, all members of a party share the same bliss point, *the party's bliss point*. Parties' bliss points are i.i.d. across parties.

The Designer sets Parliamentary institutions under the veil of ignorance: he does not know which type of issue may arise, the distribution of shares of parties' members in the population or the realized distribution of bliss points in the population or among legislators.

To formalize the Designer's information about Parliament's decision process, we assume that only one issue arises, and that the policy adopted for that issue is the weighted average of the policies that maximize the aggregate utility of each chamber, subject to some error.[4] Not modeling the Parliamentary bargaining process explicitly allows us to forgo making strong assumptions on the distribution of preferences among legislators and the Parliament Designer's knowledge of them. By definition, the weight associated with a chamber in the computation of the adopted policy is its bargaining power.

To formalize the Designer's information about the distribution of legislators across parties, we consider two allocation systems of Parliamentary seats: a proportional representation and a non-proportional representation system. In the proportional representation system, the distribution of seat shares across parties is equal to the distribution of shares of partisans in the population. In the non-proportional system, these distributions may differ. In an extension, we show that the distribution of legislators in the proportional representation system is obtained from a two-stage game. In the first stage, parties may form coalitions to maximize the shares of seats they obtain in a proportional single-district election. In the second stage, each individual casts one vote, either for a party or for a coalition of parties, to maximize his expected utility under perfect information about legislators' partisan affiliations, but imperfect information about the exact value of their bliss points.[5]

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[4] In a unicameral system, the policy adopted is the policy proposed by the unique chamber.

[5] Any game using a non-proportional voting system requires many more assumptions than the game in which voting is at-large and proportional. We would need to specify the number of districts, the size of each district (which may not be uniform in practice), the distribution of partisans in each districts, the possibility for parties to form coalitions within and across districts, etc. We do not propose such a model; instead, we make an assumption

The Parliament Designer's problem involves two tradeoffs: a tradeoff between the unit cost of a member of Parliament and the marginal benefit of having a larger Parliament to lower the risk that the policy will be decided by legislators whose preferences are far from those of the population at large, and a tradeoff between the unit cost of a chamber and the benefit of instituting a Senate to mitigate the negative impact of the error term in the adopted policy. That error term formalizes in the simplest way possible the common point in defenses of bicameralism, such as Madison's, which is that members of the Lower Chamber might not adopt the best policy according to some measure of welfare.

The model provides several predictions. First, the log of the size of Parliament increases linearly with the log of the size of the population, which is consistent with one of the two stylized facts mentioned above.

Second, the number of legislators ought not to depend on the unicameral or bicameral structure of the legislature.

Third, the relative bargaining power of a chamber in a bicameral system should optimally be equal to the share of legislators belonging to this chamber.

Fourth, countries with a larger population are more likely to have a bicameral legislature, the other stylized fact mentioned above. Fifth, the model predicts no impact of the factors that affect the size of Parliament, except the size of the population, on whether a country has a unicameral or bicameral Parliament.

We find that the number of legislators depends on whether the system is proportional or not. Our sixth prediction is that in a proportional voting system, the make-up of the partisan structure of a country does not affect the number of legislators. Seventh, in non-proportional systems, by contrast, we predict that as the partisan structure becomes more dispersed, as measured by the Herfindahl-Hirschman index, the number of legislators ought to rise. To derive testable predictions, we compute the optimal number of seats for a specific non-proportional system in which a fixed number of seats are granted to the party with the biggest share of partisans in the population, and the remaining seats are randomly distributed across all parties (including the largest party) according to their share of partisans in the population.

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on the system of allocation of seats that is consistent with an empirical analysis of non-proportional voting systems.

Empirically, we find that all these predictions are consistent with the results of a series of estimations that we conduct on a sample of 75 non-autocratic countries for which we have detailed political information over the period 1975 to 2012. For the average bargaining power of Upper Houses across bicameral countries, we use the estimation provided in Bradbury and Crain 2001.

Our model also provides specific predictions, which are borne out by the data, concerning the coefficient of the Herfindahl-Hirschman index of partisan fractionalization as well as that of other terms.

Finally, we provide two tests of the assumption we use to compute the size of Parliament in non-proportional representation systems. First, we estimate that, on average across observations at the country/party/election year level, the party ranked first by decreasing share of votes obtains around 10 percent of seats in the Lower Chamber. Second, we find that the rest of the seats are allocated across all parties (including the party ranked first) so that the share of seats and the share of votes of a party are the same, regardless of its rank.

The rest of the paper is set up as follows: Section 2 reviews related literature; Section 3 presents the model, which is used to derive the predictions exhibited in Section 4; Section 5 presents the empirical analysis; Section 6 concludes. Descriptive Statistics are in Appendix A. Appendix B presents tests of the assumptions used in non-proportional representation systems.

2 Literature Review

Our contribution relates to the literature on endogenous political institutions. The distinction between a Constitutional stage, in which decision procedures are created, and a subsequent legislative stage, in which a policy is chosen, was studied in Romer and Rosenthal 1983. While Romer and Rosenthal 1983 find a certain unanimity rule to be optimal in their setting, Aghion and Bolton 2003 find that it is optimal to require an interior majority threshold for a change in the status-quo policy.[6]

The seminal theory concerning the size of legislatures is the cube root formula proposed by

[6]The crucial difference is that Aghion and Bolton 2003 assume that there is a deadweight loss associated with implementing monetary transfers designed to compensate the losers of the reform being proposed.

Taagepera 1972, which holds that the number of legislators should equal the cube root of the population of a country. This formula is designed to prevent excessive disproportionality in representation – see e.g. Lijphart 2012. It can be theoretically motivated by a desire to minimize the number of communication channels for assembly members, who need to communicate with their constituents while also communicating with their fellow assembly members. Empirically, this formula under-predicts the sizes of Parliaments.

Auriol and Gary-Bobo 2012 adopt a mechanism-design approach to the design of Parliamentary institutions. In their model, the principal (the "Founding Fathers") does not know the distribution of preferences over a one-dimensional policy choice to be made, and knows that no agent in society will know them. Rather, the "Founding Fathers" have a diffuse prior over the set of possible distributions of preferences. Furthermore, there is an executive branch, made up of a randomly chosen single agent who is the residual claimant of all decision rights not specifically delegated to the legislative branch. The legislative branch, by contrast, is made up of n randomly chosen agents, whose role consists of revealing their preferences truthfully. Truthful revelation is effected by a VCG-mechanism applied to the agents making up the legislative branch. In our model, by contrast, there is no executive branch, and the legislators' salaries are exogenously given. This simpler model allows us to derive additional predictions concerning the structure of Parliaments, such as their bicameral or unicameral nature, the impact of different voting systems, and the effect of preferences that are homogeneous within given groups.

The literature on bicameralism has devoted some attention to the impact of a second chamber on the process of legislative bargaining and subsequent policy choices.[7]Facchini and Testa 2005 study the interaction of legislators with lobbying groups in a bicameral setting. Rogers 1998 assumes that larger chambers have lower costs for information acquisition, giving them a larger first-mover advantage.

We do not explicitly model the decision process in Parliament, which is the focus of the legislative bargaining literature, starting with Baron and Ferejohn 1989, Baron and Diermeier

[7]See e.g. Buchanan and Tullock 1962, Ch. 16, Riker 1992, Diermeier and Myerson 1994, Diermeier and Myerson 1999, Rogers 1998, Bradbury and Crain 2001, Tsebelis and Money 1997, and the references in these papers.

2001 and Diermeier and Merlo 2000. We rather assume that each chamber adopts as its policy the average bliss point of its members. This policy maximizes the sum of legislators' utilities, and thus corresponds to the behavior they would optimally adopt in a setting in which utilities were transferrable. In contrast to, for instance, the analysis in Tsebelis and Money 1997, our legislators do not anticipate the impact of their decisions on the bargaining process with the other chamber in a bicameral system.

We furthermore do not consider issues pertaining to the acquisition or transmission of information within the legislature, all legislators being perfectly informed of their respective bliss points. By contrast, an early investigation of legislators' incentives to acquire information is provided by Gersbach 1992; Austen-Smith and Riker 1987 analyze legislators' incentives to reveal or conceal private information they may have.

Our paper is also related to the literature on electoral systems. The argument most often cited in favor of proportional systems is that they lead to a more faithful representation of a population's opinions in Parliament, whereas plurality systems will be more likely to obviate the need for multi-party coalitions, thus leading to greater stability (Blais 1991, Grofman and Lijphart 1986). **?** show that countries with greater ethno-linguistic fractionalization are more likely to have plurality voting systems, which are interpreted as a means of achieving greater insulation of the political leadership. While also minimizing the role of strategic behavior in the establishment of political parties, Lijphart 1990 finds that the voting system has little impact on the number of parties. Finally, a few studies link voting systems to voter turnout (Herrera, Morelli and Nunnari 2015, Herrera, Morelli and Palfrey 2014). Our paper contributes to this literature by relating the structure of Parliament to the voting system and to partisan fractionalization.

3 Model

3.1 Set-up

We consider a population of M individuals. A Parliament Designer acts as a benevolent utilitarian who wants to maximize the representative agent's expected utility through his choice of the setup of Parliament. He chooses whether Parliament will be unicameral or bicameral, and sets the number of members of the House n_H. If he has chosen a bicameral Parliament, he also sets the number of members of the Senate n_S, and the bargaining power of the House and the Senate, denoted α and $1-\alpha$ respectively.

Preferences. An individual i's utility is:

$$u_i(x) = -(x-x_i)^2 - \frac{NC + nc}{M}$$

where $x \in \mathbb{R}$ is the policy to be adopted by the Parliament, $x_i \in \mathbb{R}$ is i's bliss point, $N \in \{1,2\}$ is the number of chambers, C the unit cost of a chamber, n the number of members of Parliament, and c the unit cost of a member of Parliament.[8]

The first term represents the payoff obtained from the policy adopted, and the second term represents the per capita contribution to the funding of Parliament.[9]

Distribution of bliss points. The population is partitioned into partisan groups or *parties* and two types of political issues may arise: *partisan* and *non-partisan*. For partisan issues, all members i of a partisan group share the same bliss point; bliss points are drawn independently across parties. For non-partisan issues, bliss points are drawn independently across individuals.

We assume that the distribution from which bliss points are drawn, either parties' bliss points

[8]The unit cost of a chamber may comprise its maintenance cost and the potential rent of the building where its members meet. The unit cost of a member of Parliament may comprise his salary.

[9]While we make the assumption of quadratic utility for the purpose of tractability, one could interpret the quadratic utility function as a second-order Taylor approximation of a more general utility function. Indeed let $u_i(x)$ be agent i's smooth utility function depending on the policy decision x. By taking the Taylor expansion of u_i at agent i's preferred policy x_i we can write

$$u_i(x) = u_i(x_i) + (x-x_i)u_i'(x_i) + \frac{(x-x_i)^2}{2}u_i''(x_i) + o((x-x_i)^2).$$

If the population taken into account by the Parliament Designer is homogeneous so that x will be close enough to x_i, the terms $o((x-x_i)^2)$ will be small enough to be neglected. As, by definition, u_i assumes its maximum at x_i, $u_i'(x_i) = 0 \geq u_i''(x_i)$. If individual i is risk averse, $u_i''(x_i) < 0$, and maximizing $\mathbb{E}[u_i]$ will be tantamount to minimizing $\mathbb{E}[-(x-x_i)^2]$. Anecdotal historical evidence suggests that Parliamentary institutions have often been ushered in by and for groups of like-minded people, which is consistent with the fact that the degree of population homogeneity is a critical determinant of the size of a nation (Alesina and Spolaore 2005).

for partisan issues or individual bliss points for non-partisan issues, is continuous and has a variance $\sigma^2 \in (0, \infty)$.

Policy adopted. Legislators have the same type of preferences as the rest of the population. Once in Parliament, legislators adopt a policy for the unique issue that has arisen. With probability $q \in [0, 1]$, the issue that arises is partisan.

If Parliament members are numbered 1 through n_H+n_S, with members of the House numbered first and members of the Senate numbered second, we assume that the policy adopted for this issue is:

$$x^* = \alpha \left(\frac{1}{n_H} \sum_{k=1}^{n_H} x_k + \tilde{Z}_H \right) + (1-\alpha) \left(\frac{1}{n_S} \sum_{k=n_H+1}^{n_H+n_S} x_k + \tilde{Z}_S \right) \tag{1}$$

where x_k is legislator k's bliss point for the issue discussed, α represents the bargaining power of the House, and $\tilde{Z}_X$ is an error term for chamber X.

The policy adopted is the weighted average of the policies that maximize the sum of the utilities of the members of each chamber, subject to some error, where the weight is the bargaining power conferred to each chamber by the Parliament Designer.

We assume that the error term $\tilde{Z}_X$ is independently drawn from a distribution of mean zero and square mean $v_X > 0$. An error term may be interpreted in various ways: it could be the probability that a subgroup of members imposes its favorite policy, or that members fail to identify their own ideal points, or that negotiations among members fail, etc.

Information. At the time of the design of Parliament, the Designer does not know which issue will arise, or the actual distribution of bliss points across individuals or across legislators. He knows that there will be G parties and has a prior belief regarding the shares of these groups in the overall population, $\boldsymbol{\gamma} = (\gamma_g)_{g=1}^G \in (0,1)^G$ (with $\sum_{g=1}^G \gamma_g = 1$). He also knows how the legislators' preferences map into the adopted policy (equation 1).

Representation in Parliament. We consider two systems of representation: a system of proportional representation in which the partisan distribution of shares of seats in Parliament is equal to the distribution of shares of partisans in the population, and a system of non-proportional representation. Legislators' bliss points for non-partisan issues are i.i.d.

3.2 Discussion of Assumptions

We take for granted that there will be a Parliament. While it is necessary for the Parliament Designer to know the variance in the distribution of bliss points, none of our calculations depend on his already knowing the mean or any other characteristics of this distribution. If the dispersion of this mean is large enough, it will be optimal to have a Parliament, rather than for the Designer to set a policy instead.

Exogenous wages c. While it may seem strange to optimize over the number of legislators given exogenous legislators' wages, we could well endogenize wages by assuming that higher wages will lead to better legislators being selected, i.e. v_X is decreasing in c. We refrain from doing so here because we do not observe v_X in the data and thus have no way of ascertaining the functional form of the dependency of v_X on c.

The population considered by the Designer may differ from the actual population. Such a case may arise for two reasons: a fixed size of Parliament may still apply many years after it was set, when the size of the population has changed, or the preferences of parts of the population, e.g. a disenfranchised population, may not be included in the Designer's objective function.

We need not interpret M as the actual population size; in fact, our predictions would continue to hold if M were a fraction or a multiple of the actual population size.

The electoral system is exogenously given. In our model, the Designer should always prefer a proportional voting system over a non-proportional system. This is because we do not integrate any offsetting benefits of non-proportional systems, which may for instance lead to greater stability of Parliamentary majorities and may favor stronger ties between legislators and their constituents, thus leading to greater accountability. In particular, we assume that there is no relationship between the voting system and the cost of Parliament or with the error terms that arise in the adopted policy of either chamber. This assumption implies that the voting system is not significantly correlated with the probability to have a second chamber; we examine this implication in Section 5.

4 Results

We first analyze the case of proportional voting, before turning to non-proportional systems. We neglect integer problems throughout our analysis.

4.1 Proportional Representation

For Proportional Representation, we assume that the shares of a party's members in the population and in Parliament are equal.

4.1.1 Unicameral Parliament

If a non-partisan issue arises, which happens with probability $(1-q) \in (0,1)$, the policy choice is

$$x^* = \frac{\sum_{d=1}^{n} x_d}{n} + \tilde{Z}_H,$$

where n is the number of legislators, and $(x_d)_{d=1}^{n}$ are their bliss points. These are i.i.d. draws from a distribution with variance σ^2.

If a partisan issue arises, the policy choice is: $x^* = \sum_{g=1}^{G} \gamma_g x_g + \tilde{Z}_H$.

The Parliament Designer maximizes:

$$\begin{aligned}\mathbb{E}[u_i] &= -\mathbb{E}[(x^* - x_i)^2] - \frac{C + n_H c}{M} \\ &= -\sigma^2 \left\{ q \left(1 - \mathbb{E}\left[\sum_{g=1}^{G} \gamma_g^2 \right] \right) + (1-q)\left(1 + \frac{1}{n_H}\right) \right\} - v_H - \frac{C + n_H c}{M}. \quad (2)\end{aligned}$$

Thus, when all parties are guaranteed proportional representation in Parliament regardless of its size, only the non-partisan issues matter for the optimal size of Parliament. Indeed, the only role of Parliament in our model consists in the sampling of population preferences. In particular, the size of Parliament will be independent of the partisan fractionalization of society. The optimal number of members of Parliament is given by:

$$n^* = n_H^* = \sigma \sqrt{(1-q)\frac{M}{c}}.$$

4.1.2 Bicameral Parliament

With two chambers, the representative agent's ex-ante expected utility is given by:

$$\mathbb{E}[u_i] = \\ -\sigma^2 \left\{ q \left(1 - \mathbb{E}\left[\sum_{g=1}^{G} \gamma_g^2\right]\right) + (1-q)\left(1 + \frac{\alpha^2}{n_H} + \frac{(1-\alpha)^2}{n_S}\right)\right\} \\ - (\alpha^2 v_H + (1-\alpha)^2 v_S) - \frac{2C + (n_H + n_S)c}{M}. \quad (3)$$

It thus follows that, in the unique candidate for an interior optimum, the relative bargaining power of a chamber equals its share of seats: $\alpha^* = \frac{n_H^*}{n_H^* + n_S^*}$. Moreover, $n_H^* = \sigma \frac{v_S}{v_H + v_S}\sqrt{(1-q)\frac{M}{c}}$ and $n_S^* = \sigma \frac{v_H}{v_H + v_S}\sqrt{(1-q)\frac{M}{c}}$, so that $\alpha^* = \frac{v_S}{v_H + v_S}$, and the overall number of members of Parliament is equal to the number of members of Parliament in a unicameral system,

$$n^* = \sigma\sqrt{(1-q)\frac{M}{c}}.$$

Finally, the difference in welfare between a bicameral and unicameral system is:

$$\frac{v_H^2}{v_H + v_S} - \frac{C}{M}$$

If v_H, v_S and C are uncorrelated with the population of a country, this difference decreases as the size of the population increases. This implies that more populous countries are more likely to have a Senate.

4.2 Extension: A Two-stage Voting Game

In a proportional voting system, the allocation of seats that we assume here is the outcome of the subgame-perfect Nash Equilibrium that is coalition-proof in the following sense.

A coalition is defined by a vector $\omega = (\theta_1, \eta_1, ..., \theta_G, \eta_G) \in (\{0,1\} \times [0,1])^G$, such that the share of seats obtained by the coalition that are attributed to party g is $\theta_g \eta_g$, with $\sum_g^G \theta_g \eta_g = 1$.[10] We

[10] If the coalition obtains a share of seats s, and hence a number of seats sn in the chamber, party g will obtain

say that g belongs to the coalition ω if $\theta_g = 1$. In other words, a coalition is defined by its members and by a contract $(\eta_1, ..., \eta_G)$ that specifies how the seats won in the chamber will be shared among them.

Let Ω denote the set of all possible coalitions, and Ω_g the set of all possible coalitions that g belongs to.

We consider the following two-stage game.

Stage 1. Suppose each party is headed by a party leader who may agree on behalf of the party to form a coalition with one or several other parties. The party leader wants to maximize the share of seats of its party.[11] In the first stage, every party g's leader chooses a unique $\omega \in \Omega_g$. Coalition ω is running in the election if and only if all parties that belong to ω have chosen it. Otherwise, we impose that all parties that picked ω run independently. Let $\overline{\Omega}(\omega_1, ..., \omega_G) \in 2^\Omega$ be the set of coalitions or parties that run in the election for a given vector $(\omega_1, ..., \omega_G)$.

Stage 2. In the second stage, every individual observes parties' choices, and casts a vote for a coalition (or a party) that is actually running. An individual i's strategy is defined by a function $\Omega^G \to \Omega$ which sets, for any vector of choices $(\omega_1, ..., \omega_G)$, which coalition (or party) $\omega \in \Omega$ individual i will vote for, and by the constraint that $\omega \in \overline{\Omega}(\omega_1, ..., \omega_G)$, i.e. that i cannot vote for a coalition that is not running in the election.

Information. At the time of the election, individuals' partisan affiliations are publicly known, but the type of issue to arise and individual bliss points are unknown. This assumption aims to account for citizens' imperfect information on the details of the issue that will come before the legislators to whom they delegate decision power. Over the course of a term, legislators may have to address unexpected issues, for instance a domestic or international crisis, or to acquire more information on an issue before deciding on a policy.

After the election, the type of issue is realized, individual bliss points are known and the policy adopted is as in Equation 1.

To simplify notations, we assume, in this section only, that the issue is partisan, that error

$sn\eta$ of these sn seats if $\theta_g = 1$, and none of them if $\theta_g = 0$.

[11]This assumption would hold in particular if the party leader is a partisan who does not sit in Parliament, which seems to be often the case empirically.

terms Z_X are null, that the Parliament is unicameral and that $C = c = 0$. The results are the same if we relax all these assumptions, and use the assumptions of the general model instead.

Lemma 1. In any subgame-perfect Nash-Equilibrium of this game, any party necessarily obtains a share of seats equal to its share of partisans in the population.

Proof. Consider the second stage first. Let i be an individual who belongs to party g, but who is not a legislator. For a partisan issue, individual i's expected utility is:

$$-\mathbb{E}\left[\left(x_g - \sum_{g'=1}^{G} \lambda_{g'} x_{g'}\right)^2\right]$$

where $\lambda_{g'}$ is the share of seats obtained by party g' in Parliament. This term can be rewritten:

$$-\sigma^2 \left[1 - \lambda_g\right]^2 - \sigma^2 \sum_{g'=1, g' \neq g}^{G} (\lambda_{g'})^2$$

This term is increasing in $\lambda_g \in [0,1]$ and decreasing in $\lambda_{g'} \in [0,1]$ for $g' \neq g$.

Let $\tilde{\lambda}'_g \geq 0$ be the share of seats obtained by party $g' \in \{1, ..., G\}$ without individual i's vote. i's vote counts for $\frac{1}{M}$ of the total share of votes, so, by definition of a proportional voting system, $\sum_{g'=1}^{G} \tilde{\lambda}_{g'} = 1 - \frac{1}{M}$.

If party g runs independently and i votes for it, i's payoff is:

$$A = -\sigma^2 \left\{ \left(1 - \tilde{\lambda}_g - \frac{1}{M}\right)^2 + \sum_{g'=1, g' \neq g}^{G} \tilde{\lambda}_{g'}^2 \right\}$$

If party g runs independently and i votes for a coalition or for a party other than g, his vote will add some $\epsilon_{g'}$ to $\lambda_{g'}$, where $0 \leq \epsilon_{g'} \leq \frac{1}{M}$ and $\sum_{g' \in G} \epsilon'_g = \frac{1}{M}$.[12] Individual i's payoff is then:

$$B = -\sigma^2 \left\{ (1 - \tilde{\lambda}_g)^2 + \sum_{g'=1, g' \neq g}^{G} (\tilde{\lambda}_{g'} + \epsilon_{g'})^2 \right\}$$

The monotonicity in $\lambda_g \in [0,1]$ and $\lambda_{g'} \in [0,1]$ implies that $A > B$. Therefore, any party that runs independently will obtain a share of seats that is at least equal to its share of partisans in the population.

[12] If i votes for a coalition, $\epsilon_{g'}$ may depend on the contract that defines that coalition.

In the first stage, no party will choose a coalition unless it can obtain at least that share of seats by doing so. Since the sum of these shares is equal to 1 no party can obtain a share of seats strictly larger than its share of partisans in any equilibrium.

Lemma 2. A situation in which all parties choose to run independently in the first stage and each individual votes for his own party in the second stage is a subgame-perfect Nash-Equilibrium.

Proof. In such a situation, a deviation by any one party will not change the set of coalitions running, since no coalition can be formed after a unique deviation.

4.3 Non-Proportional Representation

In a non-proportional representation system, the shares of a party's members in the population and among legislators may differ.

4.3.1 Unicameral Parliament

First, fix a realization of partisan shares $\boldsymbol{\gamma}$. Let k_g be the number of legislators who belong to party g. Given that a representative agent i's utility conditional on a non-partisan issue arising is given by the same expression regardless of the electoral system, we here compute his utility conditional on a partisan issue arising. We call this event $\mathcal{Q}$. This expected utility is given by

$$\begin{aligned}
\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] &= -\mathbb{E}\left[\sum_{g=1}^{G}\gamma_g\left(x_g-\sum_{g'=1}^{G}\frac{k_{g'}}{n}x_{g'}\right)^2|\boldsymbol{\gamma}\right]-v_H \\
&= -\sum_{g=1}^{G}\gamma_g\mathbb{E}\left[\left(x_g-\sum_{g'=1}^{G}\frac{k_{g'}}{n}x_{g'}\right)^2|\boldsymbol{\gamma}\right]-v_H \\
&= -\sigma^2\sum_{g=1}^{G}\gamma_g\mathbb{E}\left[1-2\frac{k_g}{n}+\sum_{g'=1}^{G}\left(\frac{k_{g'}}{n}\right)^2|\boldsymbol{\gamma}\right]-v_H,
\end{aligned}$$

where we have used that the x_g are i.i.d. draws from a distribution with a variance of σ^2. We have

$$\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] = -\sigma^2\left\{1-\frac{2}{n}\sum_{g=1}^{G}\gamma_g\mathbb{E}[k_g|\boldsymbol{\gamma}]+\frac{1}{n^2}\sum_{g=1}^{G}\mathbb{E}\left[k_g^2|\boldsymbol{\gamma}\right]\right\}-v_H.$$

We can write $k_g=\sum_{d=1}^{n}\mathbb{1}_{d,g}$, where $\mathbb{1}_{d,g}$ is an indicator equal to 1 if and only if the legislator assigned to seat d ($d \in \{1, \cdots, n\}$), is of party g, and 0 otherwise.

Thus,

$$\mathbb{E}[k_g|\boldsymbol{\gamma}] = \sum_{d=1}^{n} \mathbb{E}[\mathbb{1}_{d,g}|\boldsymbol{\gamma}] = \sum_{d=1}^{n} \Pr\left(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma}\right) = n\lambda_g(\boldsymbol{\gamma}),$$

where $\lambda_g(\boldsymbol{\gamma}) := \frac{1}{n}\sum_{d=1}^{n} \Pr\left(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma}\right)$ is the average probability (over Parliamentary seats) that a member of party g becomes a legislator.

By the same token,

$$\mathbb{E}[k_g^2|\boldsymbol{\gamma}] = \mathrm{Var}(k_g|\boldsymbol{\gamma}) + \left(\mathbb{E}[k_g|\boldsymbol{\gamma}]\right)^2 = \mathrm{Var}(k_g|\boldsymbol{\gamma}) + n^2\lambda_g^2(\boldsymbol{\gamma}).$$

Now,

$$\begin{aligned}\mathrm{Var}(k_g|\boldsymbol{\gamma}) &= \mathrm{Var}\left(\sum_{d=1}^{n} \mathbb{1}_{d,g}|\boldsymbol{\gamma}\right) \\ &=^{*} \sum_{d=1}^{n} \mathrm{Var}(\mathbb{1}_{d,g}|\boldsymbol{\gamma}) = \sum_{d=1}^{n} \left[\Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma})(1 - \Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma}))\right] = n\varsigma_g(\boldsymbol{\gamma}), \quad (4)\end{aligned}$$

where $\varsigma_g(\boldsymbol{\gamma}) := \frac{1}{n}\sum_{d=1}^{n} \left[\Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma})(1 - \Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma}))\right] = \frac{1}{n}\sum_{d=1}^{n} \mathrm{Var}(\mathbb{1}_{d,g}|\boldsymbol{\gamma})$ is the arithmetic mean over seats d of the variance of $\mathbb{1}_{d,g}$, and where we have used the assumption that the partisan affiliations of legislators are independently drawn across seats for the equality marked by *.

Thus, we have

$$\mathbb{E}[-(x^* - x_i)^2|\mathcal{Q}, \boldsymbol{\gamma}] = -\sigma^2 \left\{1 - 2\sum_{g=1}^{G} \lambda_g(\boldsymbol{\gamma})\gamma_g + \sum_{g=1}^{G} \lambda_g^2(\boldsymbol{\gamma}) + \frac{1}{n}\sum_{g=1}^{G} \varsigma_g(\boldsymbol{\gamma})\right\} - \upsilon_H.$$

Therefore, the size of Parliament matters for the representative agent's ex ante expected utility unless $\mathrm{Var}(k_g|\boldsymbol{\gamma}) = 0$ for all g, i.e. unless the composition of Parliament is deterministic, as in a proportional voting system.

In order to reduce the dimensionality of the problem of estimating $\Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma})$ for all $(d, g) \in \{1, \cdots, n\} \times \{1, \cdots, G\}$ (which would be necessary to compute the $(\varsigma_1, \cdots, \varsigma_G)$), further assumptions on the seat-allocation process are needed. We shall discuss two simple sets of assumptions below: (1) the case of an i.i.d. allocation of seats, and (2) a bonus for the biggest party.

(1) i.i.d. Allocation of Seats

In this case, $\Pr(\mathbb{1}_{d,g} = 1|\boldsymbol{\gamma}) = \lambda_g(\boldsymbol{\gamma})$ for all seats $d \in \{1, \cdots, n\}$. We shall furthermore assume that the the allocation of seats is fair in the sense that $\lambda_g(\boldsymbol{\gamma}) = \gamma_g$ for all $g \in \{1, \cdots, G\}$. In this case, the Designer's objective is given by

$$-\sigma^2 \left\{ q \left[1 - \mathbb{E}\left[\sum_{g=1}^{G} \gamma_g^2 \right] + \frac{1}{n} \left(1 - \mathbb{E}\left[\sum_{g=1}^{G} \gamma_g^2 \right] \right) \right] + (1-q) \left(1 + \frac{1}{n} \right) \right\} - v_H - \frac{C + nc}{M}.$$

Thus, the optimal number of members of Parliament is given by

$$n^* = \sigma \sqrt{\frac{M}{c}} \sqrt{q(1 - \mathbb{E}[\mathcal{H}]) + 1 - q},$$

where we write $\mathcal{H} := \sum_{g=1}^{G} \gamma_g^2$ for the Herfindahl-Hirschman index of partisan fractionalization.

(2) Bonus To The Largest Group

Without loss of generality, the group $g = 1$, also denoted Party 1, refers to the party with the largest number of members in the population at the time of the allocation of seats. If the allocation of each seat were determined by the outcome of an election in a unique district attached to that seat, if there were no coalitions formed, if individuals voted sincerely to elect members of Parliament and if the distribution of partisan affiliations were homogeneous across districts, Party 1 would win all seats in Parliament. Although all these assumptions may fail in countries that use non-proportional voting systems, the party that represents the largest share of the population may still have *some* advantage over other parties.

To formalize that advantage, we assume that party $g = 1$ automatically wins a proportion $1 - \xi$ of seats. In the other ξn seats, we make the assumption that the seat allocations are i.i.d., with the probability that a seat is allocated to party g equal to its share of partisans in the overall population. This assumption is motivated by the fact that, in practice, at the time of the design of Parliamentary institutions, the Parliament Designer may not know perfectly how individuals will migrate from one district to another, or may not even know how district boundaries will be defined, etc. We examine this assumption empirically in Appendix B.

Thus, we have $\lambda_1(\boldsymbol{\gamma}) = 1 - \xi(1 - \gamma_1)$, $\lambda_g(\boldsymbol{\gamma}) = \xi\gamma_g$ for $g \neq 1$, $\varsigma_1(\boldsymbol{\gamma}) = \xi(1 - \gamma_1)(1 - \xi(1 - \gamma_1))$, and $\varsigma_g(\boldsymbol{\gamma}) = \xi\gamma_g(1-\xi\gamma_g)$ for $g \neq 1$. Using this in our expression for $\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}]$, we find

$$\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] = \\ -\sigma^2\left\{1+(1-\xi)^2(1-2\gamma_1)-\xi(2-\xi)\sum_{g=1}^{G}\gamma_g^2 \right. \\ \left. +\frac{\xi}{n}\left[2(1-\gamma_1(1-\xi))-\xi\left(1+\sum_{g=1}^{G}\gamma_g^2\right)\right]\right\}-v_H. \quad (5)$$

Thus, the Designer's objective is given by

$$-\sigma^2\left\{q\left[1+(1-\xi)^2(1-2\mathbb{E}[\gamma_1])-\xi(2-\xi)\mathbb{E}\left[\sum_{g=1}^{G}\gamma_g^2\right] \right.\right. \\ \left. +\frac{\xi}{n}\left[2(1-\mathbb{E}[\gamma_1](1-\xi))-\xi\left(1+\mathbb{E}\left[\sum_{g=1}^{G}\gamma_g^2\right]\right)\right]\right] \\ \left. +(1-q)\left(1+\frac{1}{n_H}\right)\right\}-v_H-\frac{C+nc}{M}. \quad (6)$$

Optimizing over n gives us the optimal size of Parliament,

$$n^* = \sigma\sqrt{\frac{M}{c}}\sqrt{1-q+q\xi\left[2(1-\mathbb{E}[\gamma_1](1-\xi))-\xi\left(1+\mathbb{E}[\mathcal{H}]\right)\right]}.$$

We note that the case $\xi = 1$ corresponds to our previous i.i.d. case.

For $\xi = 0$ and given $\boldsymbol{\gamma}$, there is no uncertainty concerning the partisan composition of the legislature, as party $g = 1$ will capture all the seats. Thus, as in the case of proportional voting, the number of Parliamentary seats matters only when it comes to non-partisan issues. It is therefore no surprise that, in this case, the optimal size of Parliament corresponds to that under proportional representation.

4.3.2 Bicameral Parliament

We now examine the case of a bicameral system in a non-proportional system. The representative agent i's ex ante expected utility, conditional on a partisan issue arising, is given by

$$\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] = \\ -\sum_{g=1}^{G}\gamma_g\mathbb{E}\left[\left(x_g-\alpha\sum_{g'=1}^{G}\frac{k_{g'}^H}{n_H}x_{g'}-(1-\alpha)\sum_{g'=1}^{G}\frac{k_{g'}^S}{n_S}x_{g'}\right)^2|\boldsymbol{\gamma}\right]-\alpha^2 v_H-(1-\alpha)^2 v_S, \quad (7)$$

where k_g^X is the number of legislators of party g in chamber X.

One shows by calculations similar to those above that

$$\begin{aligned}\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] &= -\sigma^2\left\{1-2\sum_{g=1}^{G}\gamma_g\left(\frac{\alpha}{n_H}\mathbb{E}[k_g^H|\boldsymbol{\gamma}]+\frac{1-\alpha}{n_S}\mathbb{E}[k_g^S|\boldsymbol{\gamma}]\right)\right.\\ &\quad\left.+\sum_{g=1}^{G}\mathbb{E}\left[\left(\frac{\alpha}{n_H}k_g^H+\frac{1-\alpha}{n_S}k_g^S\right)^2|\boldsymbol{\gamma}\right]\right\}-\alpha^2 v_H-(1-\alpha)^2 v_S\\ &= -\sigma^2\left\{1-2\sum_{g=1}^{G}\gamma_g(\alpha\lambda_g^H(\boldsymbol{\gamma})+(1-\alpha)\lambda_g^S(\boldsymbol{\gamma}))+\sum_{g=1}^{G}\left(\alpha\lambda_g^H(\boldsymbol{\gamma})+(1-\alpha)\lambda_g^S(\boldsymbol{\gamma})\right)^2\right.\\ &\quad\left.+\sum_{g=1}^{G}\left(\frac{\alpha^2}{n_H}\varsigma_g^H(\boldsymbol{\gamma})+\frac{(1-\alpha)^2}{n_S}\varsigma_g^S(\boldsymbol{\gamma})\right)\right\}-\alpha^2 v_H-(1-\alpha)^2 v_S, \quad (8)\end{aligned}$$

where $\lambda_g^X(\boldsymbol{\gamma})$ denotes the average probability over districts that a candidate of party g is elected to chamber X, and $\varsigma_g^X(\boldsymbol{\gamma})$ is the arithmetic mean (over districts) of the variance of the random variable $\mathbb{1}_{d,g}^X$ conditional on $\boldsymbol{\gamma}$, which is 1 if a candidate of party g is elected to chamber X for the seat d, and 0 otherwise.

To make further predictions, we again analyze the i.i.d. case and the case of a bonus to the largest party, as above.

(1) i.i.d. Allocation of Seats

In this case, (8) simplifies to

$$\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] = -\sigma^2\left(1+\frac{\alpha^2}{n_H}+\frac{(1-\alpha)^2}{n_S}\right)(1-\mathcal{H})-\alpha^2 v_H-(1-\alpha)^2 v_S,$$

where $\mathcal{H} := \sum_{g=1}^{G}\gamma_g^2$ again denotes the Herfindahl-Hirschman index of partisan fractionalization.

The Parliament Designer maximizes

$$-\sigma^2\left(1+\frac{\alpha^2}{n_H}+\frac{(1-\alpha)^2}{n_S}\right)[q(1-\mathbb{E}[\mathcal{H}])+1-q]-\alpha^2 v_H-(1-\alpha)^2 v_S-\frac{2C+(n_H+n_S)c}{M},$$

and the optimum is given by

$$\alpha^*=\frac{v_S}{v_H+v_S},$$

$$n_H^*=\sigma\alpha^*\sqrt{\frac{M}{c_H}}\sqrt{q(1-\mathbb{E}[\mathcal{H}])+1-q},$$

$$n_S^*=\sigma(1-\alpha^*)\sqrt{\frac{M}{c_S}}\sqrt{q(1-\mathbb{E}[\mathcal{H}])+1-q},$$

implying

$$n^*=\sigma\sqrt{q(1-\mathbb{E}[\mathcal{H}])+1-q}\sqrt{\frac{M}{c}}, \tag{9}$$

as in the unicameral case.

(2) Bonus To The Largest Group

Straightforward calculations show

$$\begin{aligned}\mathbb{E}[-(x^*-x_i)^2|\mathcal{Q},\boldsymbol{\gamma}] &= -\sigma^2\left\{1+(1-\xi)^2(1-2\gamma_1)-\xi(2-\xi)\sum_{g=1}^{G}\gamma_g^2\right.\\ &\left.+\xi\left(\frac{\alpha^2}{n_H}+\frac{(1-\alpha)^2}{n_S}\right)\left[2(1-\gamma_1(1-\xi))-\xi\left(1+\sum_{g=1}^{G}\gamma_g^2\right)\right]\right\}-\alpha^2 v_H-(1-\alpha)^2 v_S. \end{aligned} \tag{10}$$

Thus, the Designer's objective is given by

$$\begin{aligned} &-\sigma^2\left\{q\left[1+(1-\xi)^2(1-2\mathbb{E}[\gamma_1])-\xi(2-\xi)\mathbb{E}\left[\sum_{g=1}^{G}\gamma_g^2\right]\right.\right.\\ &\left.+\xi\left(\frac{\alpha^2}{n_H}+\frac{(1-\alpha)^2}{n_S}\right)\left(2(1-\mathbb{E}[\gamma_1](1-\xi))-\xi\left(1+\mathbb{E}\left[\sum_{g=1}^{G}\gamma_g^2\right]\right)\right)\right]\\ &\left.+(1-q)\left(1+\frac{\alpha^2}{n_H}+\frac{(1-\alpha)^2}{n_S}\right)\right\}-\alpha^2 v_H-(1-\alpha)^2 v_S-\frac{2C+(n_H+n_S)c}{M}. \end{aligned} \tag{11}$$

Optimizing, we again find

$$\alpha^*=\frac{v_S}{v_S+v_H},$$

and for the optimal total number of legislators

$$n^* = \sigma\sqrt{\frac{M}{c}}\sqrt{1-q+q\xi\left[2(1-\mathbb{E}[\gamma_1](1-\xi))-\xi\left(1+\mathbb{E}[\mathcal{H}]\right)\right]}, \tag{12}$$

as in the unicameral setting. As before, $\alpha^* = \frac{n_H^*}{n^*}$, or $n_H^* = \alpha^* n^*$ and $n_S^* = (1-\alpha^*)n^*$.

Both models (1) and (2) predict that the trade-off between a unicameral and a bicameral system is the same as under a proportional voting system, implying that a bicameral system is better if and only if

$$\frac{v_H^2}{v_H + v_S} \geq \frac{C}{M}.$$

4.4 Predictions

This analysis yields the following predictions.

Prediction 1. The log of the number of legislators is increasing in the log of the size of the population, with a coefficient close to 0.5.

Prediction 2. The number of legislators is independent of whether a Parliament is unicameral or bicameral.

Prediction 3. In bicameral systems, the relative bargaining power of a given chamber is equal to the share of legislators sitting in this chamber.

Prediction 4. More populous countries are more likely to have a bicameral Parliament.

Prediction 5. The factors that impact the size of Parliament in the model (except the size of the population) have no impact on the probability that a country has a bicameral Parliament.

Prediction 6. In proportional systems, the level of partisan fractionalization has no impact on the size of Parliament. The log of the size of Parliament is:

$$\log n = \log\sigma + 0.5\log M - 0.5\log c + 0.5\log(1-q). \tag{13}$$

Prediction 7. In non-proportional voting systems, for a given q and ξ, the log-linearization of

Equation 12 implies that the log of the size of Parliament is:

$$\log n = \log \sigma + 0.5 \log M - 0.5 \log c \\ + 0.5 \log \left(1 - q + q\xi \left[2(1 - \mathbb{E}[\gamma_1](1-\xi)) - \xi \left(1 + \mathbb{E}[\mathcal{H}]\right)\right]\right). \quad (14)$$

Approximating the last term, we have:

$$\log n = \log \sigma + 0.5 \log M - 0.5 \log c + 0.5 \log(1-q) \\ - 0.5 \frac{q}{1-q} \xi^2 \mathbb{E}[\mathcal{H}] - 0.5 \frac{q}{1-q} \left(2\xi(1-\xi)\mathbb{E}[\gamma_1] + \xi^2 - 2\xi\right). \quad (15)$$

5 Empirical analysis

5.1 Methodology

This section presents various estimations that follow directly from the predictions of the model. Although the lack of exogenous sources of variations of the explanatory variables prevents proper causal identification,[13] these estimations document a certain number of stylized facts regarding the organization of Parliaments across countries.

Data. The estimations use data from the Database of Political Institutions 2012, hereafter DPI (Beck et al. 2001 [last update in 2012]). These data contain information for almost every country and every year between 1975 and 2012 about the number of chambers, the number of members of either chamber, the voting system (proportional/non-proportional), and electoral outcomes, namely vote shares and seat shares across parties, in election years for the Lower Chamber.[14] Population by country comes from the World Bank Database. To assess the "degree of democratization," we use the Polity IV score from Polity data.[15] These data indicate the Polity

[13]For instance, the partisan structure may be a strategic response to Parliamentary institutions, such as the voting system or the number of seats. Such responses may create a reverse causality bias in the estimations.

[14]There is no information on Upper Chamber elections. In fact, in many countries (e.g. in the United Kingdom or in Canada), members of the Upper Chamber are not elected.

[15]The data are available on the Polity IV website http://www.systemicpeace.org/inscrdata.html.

IV score of most countries for every year between 1800 and 2015. The Polity IV score ranges from -10 to 10, and is used to partition regimes into "Autocracies" (score between -10 and -6), and other regimes.

Sample of observations. The longest period covered by all the data sources spans 1975 to 2012 (with some missing information). Many countries have been governed by an autocratic regime for at least some years of that period. Since we consider that our model does not explain the institutional features of such regimes, we aim to exclude these observations from our sample. To do so, we include only "Non-autocratic" regimes in the Polity IV classification (i.e. we exclude countries with a score between -10 and -6 in 2012).

In addition, we exclude countries that have not had at least two legislative elections for which no party obtained all the votes and there were no reports of substantial fraud (the DPI contains a binary variable that indicates whether an election was marred by fraud).[16]

Since observations in a country across years usually cannot be considered independent – for instance, the size of the US Congress over the whole period is 535, due to a rule set in 1911 – we run all regressions on a sample of data with a unique observation by country.[17]

Definition of variables. To assess the value of the variables unrelated to electoral outcomes (the number of chambers of Parliament, the size of each chamber of Parliament, the size of the population), which are all non-random, we use the observation corresponding for 2012. To assess the values of variables related to electoral outcomes, which may be random, we compute their empirical means over all the elections (with no fraud reported and with more than one party) that took place between 1975 and 2012.

As in the model, *Party 1* refers to the party that, in a given election, obtained the largest share of the total number of votes in the country.[18]

The variables related to electoral outcomes are the Herfindahl-Hirschman index of partisan

[16]This restriction excludes countries that may have had more than one election, but for which there is no electoral information.

[17]Including many years of observations would artificially increase the significance of the impact of any variable that changes little between 1975 and 2012, such as the size of the Parliament, the population, the number of chambers, etc.

[18]The identity of Party 1 may of course change from one election to the next.

fractionalization $\mathcal{H} = \sum_g \gamma_g^2$, the share of total votes obtained by Party 1 γ_1, and ξ. The actual value of $\mathcal{H}$ and γ_1 for any democratic election that took place between 1975 and 2012 can be computed directly from the data, and we use their empirical means over democratic elections as proxy variables for their expected values.[19]

Conversely, the actual value of ξ cannot be obtained directly from any variable in the data. In fact $1-\xi$, which we informally refer to as "the bonus to the largest party," may depend on many institutional features – such as redistricting rules – that cannot be easily quantified. Instead, to assess the value of ξ, we use the fact that, by definition, the expected proportion of seats won by Party 1, $\mathbb{E}[\lambda^H]$, is $\mathbb{E}[1-\xi+\xi\gamma_1]$. The proxy variable for ξ is then $\frac{1-\mathbb{E}[\lambda^H]}{1-\mathbb{E}[\gamma_1]}$, where we use averages across elections to estimate the expectations in the equation.[20][21]

Appendix A reports the descriptive statistics of the variables used in the empirical analysis.

Specification. The basic specification for the estimations is:

$$\log n_k = \beta_0 + \beta_1 \log M_k + \beta_2 \xi_k^2 \mathbb{E}[\mathcal{H}]_k + \beta_3 \Big(2\xi_k(1-\xi_k)\mathbb{E}[\gamma_1]_k + \xi_k^2 - 2\xi_k \Big) + \epsilon_k \tag{16}$$

with one observation by country k, and where M_k is the size of the population in 2012, $\mathbb{E}[\mathcal{H}]_k$ is the average Herfindahl-Hirschman index across democratic elections in country k, $\mathbb{E}[\gamma_1]_k$ is the average share of votes obtained by Party 1 in country k across democratic elections, ξ_k is the ratio of one minus the average share of seats obtained by Party 1 across democratic elections over one minus the average share of votes obtained by Party 1 across democratic elections in country k. The dependent variable $\log n_k$ is the log of the total number of members of Parliament in country k in 2012.

[19] In certain cases, the vote shares of some parties may be either missing or not consistent. Since precise information on vote shares is crucial to compute the Herfindahl-Hirschman index, we include only observations such that the sum of the known shares of the parties is between 0.90 and 1.10.

[20] The model assumes that ξ is non-random. If it were random, the relationship $\mathbb{E}[\xi\gamma_1 + 1 - \xi] = \mathbb{E}[seats_1]$ still holds, but cannot be used to derive $\mathbb{E}[\xi]$ in general if ξ and γ_1 are not independent. In fact, there would be no general proxy variable for the expected values of the polynomials of ξ and the shares of votes that appear in the model if we didn't assume that ξ and these shares were independent.

[21] We note that the maximum value of ξ is larger than 1, which is due to the fact that, in a few cases, the party that obtained the largest share of votes got fewer seats in Parliament.

Equation 16 does not include variables for which there is no information (c) or that have no obvious empirical proxy (q and σ).

Predictions 6 and 7 imply that:

- Under any voting system, the model predicts $\beta_1 = 0.5$.

- Under a proportional voting system, ξ is equal to 1, $\beta_2 = 0$. Term (3) and the intercept would be collinear in theory, yet in practice, if $\xi_k = 1 + u_k$ where u_k is some random error of mean 0 – due for instance to integer problems in the attribution of seats – we may have $\beta_3 = 0$.

- Under a non-proportional voting system with an i.i.d. distribution of seats across parties, ξ is equal to 1, $\beta_2 = -0.5\frac{q}{1-q}$, which implies that $\beta_2 < 0$. Term (3) and the intercept would be collinear in theory, yet in practice, as under a proportional voting system, we may have $\beta_3 = 0$.

- Under a non-proportional voting system that gives a bonus of seats to Party 1, $\beta_2 = -0.5\frac{q}{1-q}$ and $\beta_3 = -0.5\frac{q}{1-q}$, which implies that $\beta_2 < 0, \beta_3 < 0$ and $\beta_2 = \beta_3$.

5.2 Results of the estimations

Table 1 reports the estimation of the coefficients of Equation 16 for the sample of non-autocratic regimes.

Table 1: **Size of Parliaments in non-autocratic regimes**

	Log Size of Parliament							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Population	0.417***	0.407***	0.408***	0.402***	0.353***	0.351***	0.427***	0.411***
	(0.023)	(0.024)	(0.023)	(0.023)	(0.040)	(0.041)	(0.030)	(0.032)
Herfindahl-Hirschman $\mathbb{E}[\mathcal{H}]$			-0.875**		-0.505		-1.130**	
			(0.376)		(0.585)		(0.542)	
$\xi^2 \times \mathbb{E}[\mathcal{H}]$				-0.833**		-0.548		-1.127**
				(0.328)		(0.594)		(0.476)
$2 \times \xi \times (1-\xi) \times \mathbb{E}[\gamma 1] + \xi^2 - 2 \times \xi$				-0.922***		0.060		-1.219**
				(0.342)		(1.026)		(0.495)
Bicameral Parliament		0.144						
		(0.092)						
Voting System	*Any*	*Any*	*Any*	*Any*	*Proportional*	*Proportional*	*Non-Proportional*	*Non-Proportional*
# Observations	75	75	75	75	33	33	42	42
R2	0.813	0.820	0.827	0.831	0.729	0.732	0.865	0.871
Wald p-value $H0 : \beta 2 = \beta 3$								.482

NOTES. This table reports the estimation of the regression of the log of the total number of members of Parliament on the log of Population and other covariates. There is one observation by country. The sample comprises all countries that were not autocratic in 2012 in the Polity IV classification and that have had at least two elections with no party getting all the votes and without reported fraud between 1975 and 2012 (see Section 5 for details). Standard errors in parentheses. $*p < 0.05, **p < 0.01, ***p < 0.001$.

Column 1 shows that the log-linear relationship from the model holds, with a coefficient close to 0.5, which is consistent with Prediction 1.

In Column 2, we include as a covariate a binary variable equal to 1 if and only if country k has a bicameral system. This variable has no significant impact on the log of the size of Parliament, which is consistent with Prediction 2.

Columns 3 and 4 of Table 1 show that the coefficients of the Herfindahl-Hirschman index (Column 3), or of the second and third terms of Equation 16 (Columns 4) have the sign predicted by the model for a non-proportional voting system. These results are consistent with the fact that the coefficients are each some average of the coefficient in the proportional voting system (which is null) and in the non-proportional system (which is negative).

In fact, if we restrict the sample to countries with a Proportional voting system (Columns 5 and 6), we find no statistically significant impact of the Herfindahl-Hirschman index, or of the coefficients of the second and third terms of Equation 16, which is consistent with Prediction 6.

If we then restrict the sample to countries with a non-proportional voting system (Columns 7 and 8), we find a negative and statistically significant impact of the Herfindahl-Hirschman index, as well as of the coefficients of the second term of Equation 16, which is consistent with Prediction 7.

Column 8 also shows that the coefficient of the third term is negative, statistically significant, and close to the value of the coefficient of the second term. The p-value of a Wald test of the hypothesis H0: $\beta_2 = \beta_3$, reported at the bottom of Column 8, shows that we cannot reject the possibility that these coefficients are equal at the 0.05 (or 0.1) level of significance. These results are consistent with Prediction 7 in the case of a "bonus to the largest group".

Remark. These results may be used to assess the probability q that a partisan issue arises, a parameter that has no obvious measurable equivalent. We have here that $0.5\frac{q}{1-q}$ is around 1.2, so that q is around 70 percent.

To assess Prediction 3, we rely on Bradbury and Crain (2001) who provide the only estimation of the ratio in the bargaining powers of the Lower and the Upper Chamber across countries, i.e. $\frac{\alpha^*}{1-\alpha^*}$ with the previous notations. They find that the ratio of bargaining powers is 3.5 on average. This means that α^* is around 0.78 on average across countries. With our data on the sizes of Lower and Upper Chambers, we estimate that the ratio of the number of members of the Lower Chamber over the total number of members of a Parliament is equal to 0.73 on average among non-autocratic regimes. This average is very close to the estimate of Bradbury and Crain (2001).[22]

Table 2 examines the determinants of an Upper Chamber.

[22] If we restrict the sample to the sample of countries used in Bradbury and Crain (2001)'s estimations, the share is 0.74.

Table 2: **Probability to have a bicameral Parliament**

	Bicameral Parliament			
	(1)	(2)	(3)	(4)
Log Population	0.071**	0.066**	0.077**	0.083***
	(0.029)	(0.030)	(0.030)	(0.030)
Proportional voting		-0.145		
		(0.114)		
Herfindahl-Hirschman $\mathbb{E}[\mathcal{H}]$			0.540	
			(0.488)	
$\xi^2 \times \mathbb{E}[\mathcal{H}]$				0.485
				(0.426)
$2 \times \xi \times (1-\xi) \times \mathbb{E}[\gamma 1] + \xi^2 - 2 \times \xi$				0.670
				(0.444)
# Observations	75	75	75	75
R2	0.074	0.095	0.090	0.110

NOTES. This table reports the estimation of the regression of a binary variable equal to 1 if and only if the country has a bicameral Parliament, on the log of Population and other covariates. There is one observation by country. The sample comprises all countries that were not autocratic in 2012 in the Polity IV classification and that have had at least two elections with no party getting all the votes and without reported fraud between 1975 and 2012 (see Section 5 for details). See Section 5 for details. Standard errors in parentheses. $*p < 0.05, **p < 0.01, ***p < 0.001$.

In Column 1, we regress a binary variable equal to 1 if and only if the country has a Bicameral Parliament on the log of the population of a country. We find that larger countries are significantly more likely to have a Senate, which is consistent with Prediction 4.

In Column 2, we include a covariate equal to 1 if and only if the country has a Proportional voting system. This variable is not significantly correlated with the dependent variable, which is consistent with Prediction 5.

In Columns 3 and 4, we find no impact of the other terms considered before.

6 Conclusion

We have analyzed a simple model of Parliamentary institutions, testing the consistency of its predictions with cross-country data. We have seen that the size of a country's Parliament increases in a log-linear manner with the size of its population. The size of a country's Parliament does not depend on whether it is unicameral or bicameral. In bicameral systems, the relative weight of a chamber should correspond to the share of legislators sitting in this chamber. Furthermore, the only observable variable impacting the probability that a given country has a bicameral Parliament is the size of its population.

A second set of results pertains to the impact of partisan fractionalization and voting systems on Parliament size. We find that the mode of election has no impact on the probability that a given country has a second chamber, and that greater fractionalization increases the size of a country's Parliament if and only if it has a non-proportional voting system. In proportional systems, fractionalization has no impact.

Our model could be extended in several ways. For instance, we do not model the bargaining process among legislators, assuming instead that legislators within a given chamber act cooperatively and do not take the bargaining process with the other chamber into account. Whether a richer model of Parliamentary procedure would preserve, or possibly even increase, the predictive power of our model is an interesting question for future research.

Furthermore, we have analyzed only cases in which the mode of election for both chambers of Parliament is the same. In reality, many countries use different modes of election for the two chambers of their Parliaments. Indeed, by extending our model, one can see that, optimally, the errors should be as negatively correlated as possible between chambers; arguably, one way of effecting a negative correlation would consist in having different voting systems for the two chambers. We recommend for future research a more detailed investigation of differences between chambers in bicameral systems.

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Appendix A. Descriptive Statistics

Table 3: Descriptive Statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Population in million	44.71	149.53	0.11	1236.69	75
Size of Parliament	250.63	220.55	28	955	75
Size of Lower Chamber/House	209.88	171.31	15	650	75
Size of Upper Chamber/Senate	87.31	82.25	11	326	35
Bicameral Parliament	0.47	0.5	0	1	75
Proportional voting system	0.44	0.5	0	1	75
Share of legislators in Lower Chamber	0.73	0.09	0.54	0.91	35
Herfindahl-Hirschman index $\mathcal{H}$	0.37	0.12	0.17	0.83	75
ξ	0.93	0.2	0.48	2	75
$\xi^2 \times \mathcal{H}$	0.34	0.37	0.12	3.31	75
$2 \times \xi \times (1-\xi) \times \gamma_1 + \xi^2 - 2 \times \xi$	-0.95	0.35	-3.63	-0.39	75

SOURCES: DPI 2012.

NOTES: This table reports descriptive statistics for all non-autocratic countries in 2012. All variables are estimated in 2012, except the electoral outcomes terms $\mathcal{H}$, ξ, $\xi^2 \times \mathcal{H}$ and $2 \times \xi \times (1-\xi) \times \gamma_1 + \xi^2 - 2 \times \xi$, which are the empirical means of these terms over legislative elections that took place between 1975 and 2012. The number of members in a chamber may differ from the number of seats in that chamber if some seats are not taken up. DPI does not give information on the size of the United Kingdom's House of Lords after 1997. If we assign the value 821 to the number of Lords in 2012 (from the House of Lords annual report for 2012-2103), the results of the estimations are similar.

Appendix B. Empirical examination of the *Bonus to the largest group* assumption

In this section, we present a series of estimations that use data of the DPI at the election and party level, for any election that took place between 1975 and 2012.

To do so, we use the data's detailed information on shares of seats and votes obtained by the largest parties across elections to test the main implication of this assumption, namely that there exists ξ such that:

$$\mathbb{E}[\lambda_g] = \xi \times \mathbb{E}[\gamma_g] + (1 - \xi) \times \mathbb{1}_{g=1} \quad (i)$$

with the notations of the model, and where $\mathbb{1}_{g=1}$ is a binary variable equal to 1 if and only if $g = 1$.

As mentioned in Section 5, we do not observe ξ directly, but infer its value from solving equation (i) with $g = 1$. Empirically, we find that, in non-proportional voting systems, ξ is equal to 0.89 on average (whereas it is equal to 0.98 on average in proportional systems). This estimation indicates that countries that use non-proportional voting systems indeed give an advantage to the party with the largest share of votes in the country, which amounts to 11% of House seats on average.

Our estimation method for ξ implies that equation (i) is trivially satisfied for Party 1 empirically. It is also trivially satisfied empirically in elections with only two parties. We therefore focus on estimating the relation between the shares of seats and votes for parties $g > 1$ only in elections with three parties or more.

The basic specification we use here is:

$$Seats\ Share_{k,g,t} = \delta_0 + \delta_1\ \xi_k \times Votes\ Share_{k,g,t} + \sum_{r=2}^{G-1} \delta_r\ Party\ rank = r_{k,g,t} + \epsilon_{k,g,t} \qquad (17)$$

where $Seats\ Share_{k,g,t}$ is the share of seats and $Votes\ Share_{k,g,t}$ is the share of total votes obtained by party g in country k for the election that took place in year t, ξ_k is defined as before for country k, and $Party\ rank = r_{k,g,t}$ is an indicator variable equal to 1 if and only if party g is ranked r^{th} in decreasing order of vote shares in country k for the election that took place in year t. We also include as covariates the fixed effects of any country/election year. (The $Party\ rank = G$ variable is excluded to avoid collinearity with the constant term. The party with the smallest share of votes is thus the reference group). Standard errors are clustered by country to account for the fact that error terms are correlated within countries.

Table 4 reports the estimation of this regression separately for all elections with a given number of parties running – from 3 to 8 parties – and for all elections.[23][24]

[23]The data do not give information on more than 9 parties, so that they may aggregate both votes and seats for elections with more than 8 parties. By definition, including elections with more than 8 parties would create a measurement error that would bias the estimations of the coefficients.

[24]The electoral system of a country may give an advantage or a disadvantage to the smallest party instead of a party of some given rank r. Reporting estimations on subsamples of observations partitioned by number of parties allows to check whether this is the case indeed. There are not enough observations by country to run the regression separately for every country, which would provide an even stronger test of the assumption.

Table 4: **Distribution of seat shares in non-proportional voting systems**

	Seats share						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\xi\times$ Votes Share	0.815***	0.943***	1.095***	0.936***	1.299***	1.170***	0.979***
	(0.147)	(0.146)	(0.104)	(0.176)	(0.231)	(0.071)	(0.069)
Party rank = 2	0.086	0.047	0.018	0.033	-0.012	-0.011	0.039
	(0.065)	(0.043)	(0.030)	(0.061)	(0.039)	(0.016)	(0.023)
Party rank = 3		-0.022	-0.034*	-0.003	-0.035	-0.029	-0.016
		(0.013)	(0.016)	(0.013)	(0.024)	(0.018)	(0.010)
Party rank = 4			0.001	-0.007	-0.045*	-0.022	-0.003
			(0.009)	(0.009)	(0.021)	(0.015)	(0.008)
Party rank = 5				0.009	-0.029*	-0.020*	-0.002
				(0.013)	(0.016)	(0.010)	(0.007)
Party rank = 6					-0.015	-0.005	-0.002
					(0.008)	(0.007)	(0.006)
Party rank = 7						-0.004	-0.000
						(0.002)	(0.005)
# *Parties*	3	4	5	6	7	8	3 to 8
# Observations	74	84	208	100	90	70	674
R2	0.928	0.900	0.898	0.865	0.926	0.961	0.923
Wald p-value $H0: \delta = 1$	.231	.702	.371	.723	.228	.060	.761

NOTES. This table reports the estimation of the regression 17. The dependent variable is the share of seats in the House that party g in country k obtained after the election in year t. There is one observation by country, party and election year. All estimations include a fixed effect for any country and election-year pair. The sample comprises all countries that were not autocratic in 2012 in the Polity IV classification and that have had at least two elections with no party getting all the votes and without reported fraud between 1975 and 2012 (see Section 5 for details). Standard errors clustered at the country level are in parentheses. $*p < 0.05, **p < 0.01, ***p < 0.001$.

These estimations show first that the coefficient of the term $\xi_k \times Votes\ Share_{k,g,t}$ is significantly different from 0, and close to 1. In fact, the p-value of a Wald test indicates that we cannot reject the hypothesis that the coefficient of $\xi_k \times Votes\ Share_{k,g,t}$ is equal to 1. This result is consistent with the equation (i) above.

The estimations also show that most indicator variables $Party\ rank = r_{k,g,t}$ have no significant effect at the 5 percent level, and none when all observations are included (Column 7). This result means that a party that obtains a share of votes such that it would be of rank r has no significant advantage or disadvantage in terms of Parliamentary seats (with respect to the party with the largest rank, which is the reference group in these estimations). On average across countries, we thus cannot reject at the 5 percent level the hypothesis that the share of votes and the share of seats (deflated by the factor ξ), are the same.